THE UNIVERSITY OF ALBERTA

STUDY OF A (d,n) REACTION

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled Study of a (d,n) Reaction, submitted by Jean-Pierre Bernier, in partial fulfilment of the requirements for the degree of Master of Science.

ABSTRACT

While deuteron stripping theory has very successfully explained experimental results of most (d,n) and (d,p) reactions, there are some instances of marked disagreement with the simpler forms of the theory. In particular, backward peaks in the angular distribution of the reaction products sometimes appear; an explanation of these based on an exchange effect called heavy-particle stripping has been put forward by Owen and Madansky. This explanation is sound in principle, but involves an over-abundance of adjustable parameters and ancillary assumptions which make detailed verification difficult and ambiguous.

The present work contains (1) an exposition of some details of the basic theory, designed to make what follows intelligible; (2) a critical examination of work published by the proponents of heavy-particle stripping concerning the reaction most extensively studied, $B^{11}(d,n)C^{12*}$ 4.43 Mev $(\gamma)C^{12}$, including the recent extension of this work to neutron-gamma angular correlations; and (3) an account of an attempted alternative explanation of the data by a new approximate method of intruducing an effect of Coulomb forces into (d,n) stripping theory; this attempt does not succeed in explaining the B^{11} (d,n) data, but the method is of some interest in itself. Other possible explanations are also mentioned.

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Chapter 1 GENERAL INTRODUCTION

A- Description of the Stripping Process

To help visualize the nature of reaction mechanisms discussed in this thesis, a brief description of the essential features of the deuteron stripping process is presented here in qualitative and pictorial terms. More extensive discussion along this line can be found in the first chapter of Butler's monograph (1).

The usual description of nuclear reactions in terms of the compound nucleus hypothesis, originally due to Niels Bohr, has very successfully described most but not all nuclear reactions. It pictures the reaction as occurring in two stages. The first stage is the amalgamation of the whole incident projectile into the target nucleus to form a compound nucleus, not directly observed, and generally in a highly excited state. After a time very long compared with the time it would have taken the projectile simply to traverse target dimensions had there been no interaction, the second stage of the reaction, namely the break-up or decay of the compound nucleus into outgoing particle and residual nucleus, takes place according to laws independent of the first stage. The compound nucleus retains no memory of its particular mode of formation, the energy and momentum of the incoming projectile being very quickly shared with all nuclear constituents. Its decay is largely akin to evaporation of a molecule from a liquid drop, governed entirely by statistical

laws. There is thus no reason why reaction products should come out in a direction close to the incident beam direction, and indeed a more thorough investigation predicts symmetry between the forward and backward hemispheres to characterize the angular distribution of reaction pruducts.

Entirely different are the more recently studied processes of direct nuclear reaction, of which deuteron stripping is the best-known instance. There is no compound nucleus. The projectile usually makes a grazing collision with the target — otherwise absorption into a compound nucleus is more likely to occur — and interacts only during the short time it takes to pass the target at something like its initial speed. As a result the outgoing particle is the projectile itself, plus or minus a few things lost to or picked up from the target, and its direction is likely to be close to the incident beam direction.

The deuteron is a rather loosely bound composite structure in which the constituent neutron and proton can sometimes be found separated by relatively large distances. Because of this a deuteron incident upon a nuclear target can with appreciable probability make such a collision that only one of its two constituent nucleons actually hits the target and is absorbed, while the other misses it entirely and sails on, more or less undisturbed, with whatever momentum direction it had at the instant of collision, to be counted as a reaction product. (Figure 1) It is already obvious from this simple description that there should be many more such reaction products emitted in directions not too different from the original beam direction

Figure 1 Deuteron Stripping

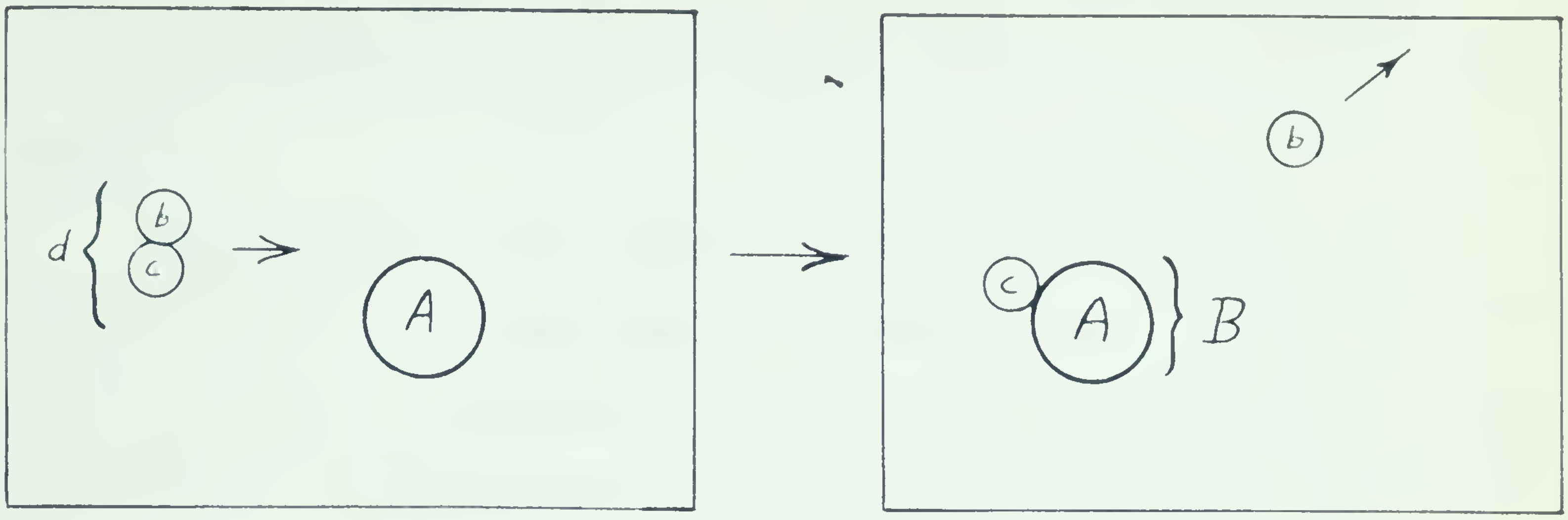


Figure 2 The Momentum Balance in Deuteron Stripping

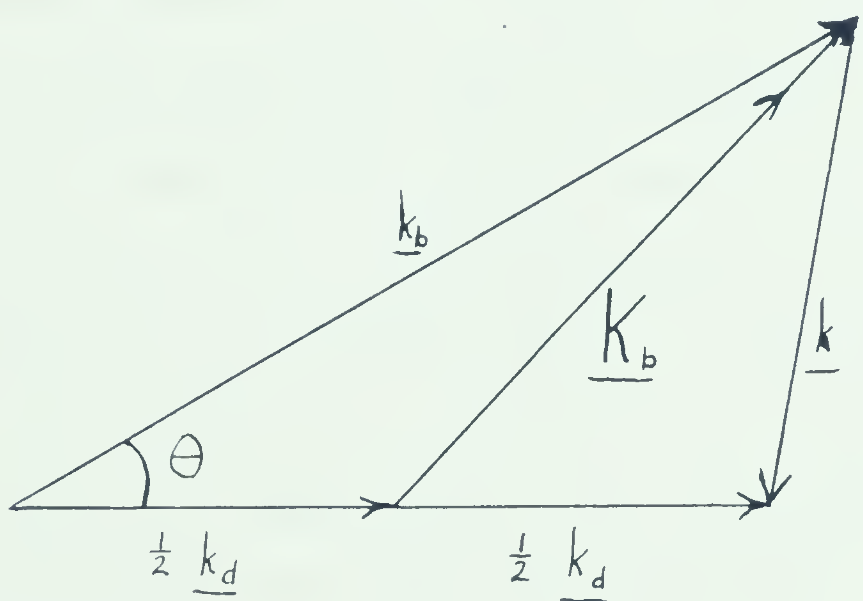
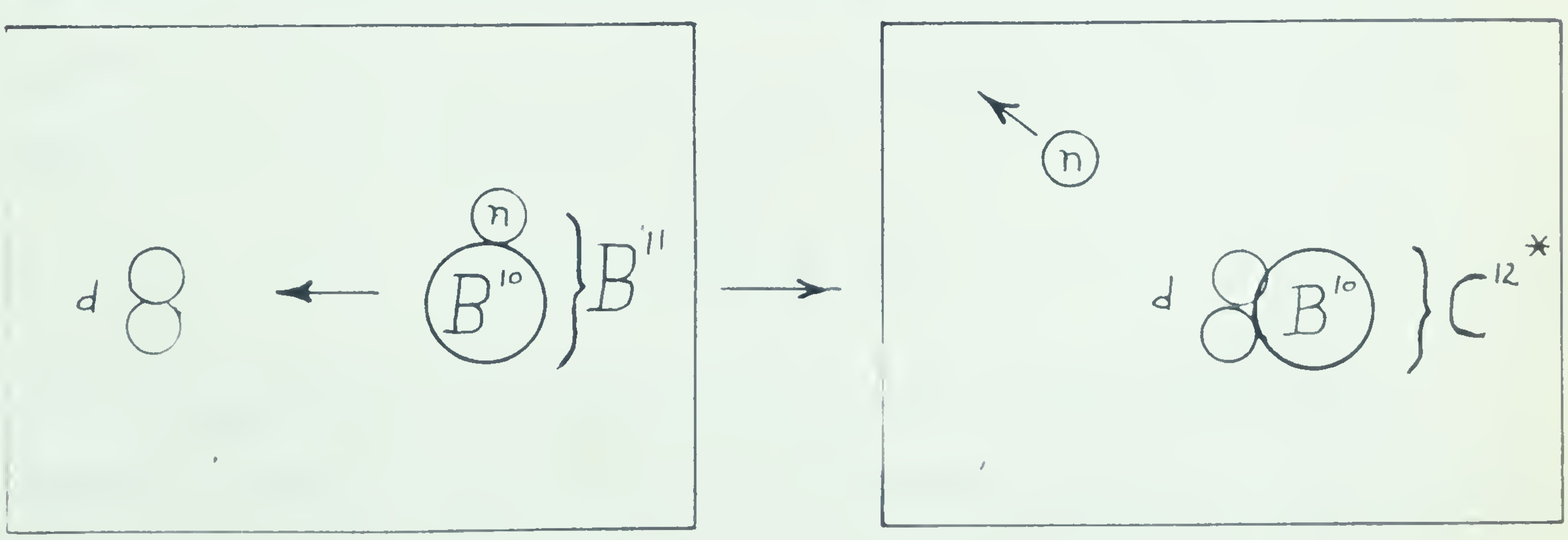


Figure 3 Heavy-Particle Stripping



than in the backward direction. This will be confirmed by detailed evaluation of the stripping cross-section in the next section.

Whether or not the nucleon which hits the target is absorbed will be determined to a large extent by selection rules corresponding to conservation of angular momentum and of parity. A classical interpretation of this angular momentum immediately yields more information about the angular distribution. The change in angular momentum between the initial and final nuclei must be provided by the total angular momentum with which the captured nucleon comes into the target; this in turn must be compounded of that nucleon's spin, of magnitude $\frac{1}{2}$, and of its orbital angular momentum ℓ_c with respect to the target nucleus. This fact restricts ℓ_c to very few possible values; conservation of parity restricts it further, to odd values only if the initial and final nuclei have different parity, to even values only if they have the same parity. The shape of the angular distribution depends very critically on these few allowed values of ℓ_c — in practice on the smallest of them — as we can see classically: ℓ_c must equal kb, where k is the relative momentum of captured nucleon approaching target, and b is the impact parameter, which can not exceed the nuclear radius R if the nucleon is to be absorbed. Thus

$$k = \frac{\ell_c}{b} \geqslant \frac{\ell_c}{R}.$$

Now the momentum k is provided in part by the deuteron's external motion, in part by the internal motion of the nucleon

while bound in the deuteron, so that the probability of such momentum being available decreases sharply with k. We have a fairly good idea therefore of the shape of the cross-section curve as a function of k: if $\ell_c = 0$ is allowed, a peak occurs at $k = 0$ with smaller peaks for higher allowed ℓ_c values; if $\ell_c = 0$ is not allowed, a peak occurs at $k = 1/R$ times the smallest ℓ_c value allowed, with further smaller peaks for higher ℓ_c.

But it is easy to see from the internal momentum balance in the deuteron's own center of mass that, if θ is the angle between incident beam and outgoing reaction product, small θ corresponds to small k and large θ to large k (Figure 2): for if, as is most probable, the outgoing nucleon stays close to the deuteron beam direction (small θ), then k gets little contribution from the deuteron's internal motion; whereas if the outgoing nucleon is to turn around completely (large θ), the captured nucleon must get a big push forward (hence large k) to conserve momentum in the deuteron's center of mass. Thus the qualitative description of the cross-section curve as a function of k given above, also describes the angular distribution of reaction products as a function of θ . The calculation outlined below, as well as other more elaborate ones, confirm these general features.

B- The Born Approximation Derivation

Similar formulas for the angular distribution have been obtained by Butler (2), using an elaborate mathematical technique to match wave functions and their derivatives at the nuclear boundary, and by Bhatia et al. (3), using the Born approximation. Later and more detailed investigations have established that Butler's approach, while sounder in principle, is in fact equivalent to a Born approximation; that while a Born approximation is not really valid here in the usual sense, there are reasons which condone its use; and that either formula accounts strikingly well for most experimental results, provided one is allowed to treat the nuclear radius R as an adjustable parameter to compensate for the approximations made. These facts will be taken for granted without discussion in this thesis.

The derivation of Bhatia et al. is much simpler than any other, leads to a formula which can be conveniently interpreted, and is the one upon which the work reviewed in Chapter 2 has been based. The outline presented here follows the useful review article by Huby (4), as well as the original paper of Bhatia et al. (3).

The basic approximations common to all treatments must be clearly understood. One neglects entirely nuclear interactions between the deuteron and the target nucleus before stripping, as well as between the outgoing nucleon and the residual nucleus, and between the neutron and the proton from the

deuteron after stripping. All Coulomb effects are likewise neglected in the basic treatment; their subsequent introduction is discussed in Chapter 3. Under these assumptions, products of plane waves and appropriate internal wave functions can be used for the initial and final wave functions to be inserted into the Born formula. The basic idea of this treatment — and of others — is that the deuteron is broken up instantaneously and its momentum balance at that instant determines subsequent happenings: the corresponding momentum of the outgoing nucleon is the momentum actually observed, which defines θ ; and in effect, the captured nucleon is then pictured as approaching the target nucleus as a plane wave with the momentum it obtains from that break-up.

The formal derivation is presented in this section, and the significance of the various factors is discussed in the next. Consider the reaction A(d,b)B (Figure 1), whereby deuteron bombardment of an initial nucleus A, with spin J_A, leaves the residual nucleus B in a particular state of spin J_B. The subscript c will indicate quantities belonging to the nucleon c which is captured (e.g. the proton in a (d,n) reaction).

According to the Born approximation, the cross-section for the reaction to proceed from initial magnetic substates M_A of A and μ_d of the deuteron to final substates M_B of B and μ_b of the outgoing nucleon is found to be

$$\sigma\,(M_A \mu_d M_B \mu_b) = \frac{m_b^{*} m_d^{*} k_b}{4\pi^2 \hbar^4 k_d} \left| T \right|^2, \qquad \underline{B1}$$

where B2

$$T = \int \chi^{*}_{M_B}(\underline{r_c}, \sigma_c, \xi)\, \chi^{*}_{b}(\sigma_b)\, e^{-i\underline{k_b}\cdot\underline{r_b}}\, V(\underline{r_c}, \sigma_c, \xi)\, \chi^{o}_{M_A}(\xi)\, \chi_d(\underline{r_b}-\underline{r_c}, \sigma_c, \sigma_b)\, e^{i\underline{k_d}\cdot(\underline{r_c}+\underline{r_b})/2}\, d\underline{r_c}\, d\underline{r_b}\, d\sigma_c\, d\sigma_b\, d\xi$$

This expression gives the cross-section in the center-of-mass coordinates provided the mass symbols are given the following meanings:

$$m_d^* = \frac{m_d m_A}{m_d + m_A}\ ; \qquad m_b^* = \frac{m_b m_B}{m_b + m_B}\ ; \qquad m_c^* = \frac{m_c m_A}{m_c + m_A}\ .$$

$\underline{k_d}$ is the initial wave number of relative motion of deuteron with respect to A, so that the total kinetic energy before collision in the center-of-mass system is

$$\frac{\hbar^2\, k_d^2}{2\, m_d^*}\ .$$

Similarly, $\underline{k_b}$ is the wave number of relative motion of b with respect to B after collision, the corresponding kinetic energy being given by

$$\frac{\hbar^2\, k_b^2}{2\, m_b^*}$$

in the center-of-mass frame. χ_d, $\chi^{o}_{M_A}$, χ_{M_B} are internal wave functions of the deuteron, target nucleus A (ground state), and residual nucleus B, respectively. $V(\underline{r_c}, \sigma_c, \xi)$ is the interaction potential between the nucleons in A, whose coordinates are collectively denoted by ξ, and the captured nucleon c. And $\underline{r_c}$, σ_c, $\underline{r_b}$, σ_b, etc. are respectively the position and spin coordinates of the particle designated by the subscript.

We pick out first from B2 the integral over $d\xi$,

$$T' = \int \chi^{*}_{M_B}(\underline{r_c}, \sigma_c, \xi)\, V(\underline{r_c}, \sigma_c, \xi)\, \chi^{o}_{M_A}(\xi)\, d\xi ,$$

which is a function of $\underline{r_c}$ and of σ_c, and assume that the capture takes place at the radius $r_c = R$, so that we write

$$T' = \frac{\delta(r_c - R)}{R^2}\, T''$$

where T" is a function of θ_c, ϕ_c, and σ_c but is independent of r_c, and may thus be expanded as follows:

$$T'' = \sum_{l_c m_c \mu_c} \langle M_B | V | M_A, l_c, m_c, \mu_c \rangle\, Y_{l_c}^{m_c *}(\theta_c \phi_c)\, \chi^{*}_{\mu_c}(\sigma_c) .$$

The Dirac brackets $\langle\ |\ |\ \rangle$ are numerical coefficients giving the projection of T" on the product function $Y_{l_c}^{m_c} \chi_{\mu_c}$.

The $Y_{l_c}^{m_c}$ are the usual spherical harmonics and the χ_{μ_c} are eigenfunctions of a spin ½ particle with components μ_c along the axis of quantization.

Substituting back one finds

$$T = \sum_{l_c m_c \mu_c} \langle M_B | V | M_A, l_c, m_c, \mu_c \rangle \langle l_c, m_c, \mu_c, b | d \rangle \qquad \underline{B3}$$

where

$$\langle l_c, m_c, \mu_c, b | d \rangle = \int Y_{l_c}^{m_c *}(\theta_c, \phi_c)\, \chi^{*}_{\mu_c}(\sigma_c)\, \chi^{*}_{b}(\sigma_b)\, e^{-i\underline{k_b}\cdot\underline{r_b}} \times$$
$$\times \chi_d(\underline{r_b} - \underline{R}, \sigma_c, \sigma_b)\, e^{i\underline{k_d}\cdot(\underline{R}+\underline{r_b})/2}\, d\underline{r_b}\, d\sigma_c\, d\sigma_b\, d\omega_c \qquad \underline{B4}$$

The symbol $\underline{R}$ denotes the vector whose spherical polar coordinates are $r_c = R$, θ_c, ϕ_c. The element of solid angle $\sin\theta_c\, d\theta_c\, d\phi_c$ is written $d\omega_c$.

The expression discussed above for $|T|^2$ should really be summed over final and averaged over initial magnetic quantum numbers μ_b, μ_d, M_A, and M_B, since these quantities are experimentally unspecified: the various final substates all contribute incoherently, and the incident beam and target nuclei are unpolarized. Hence

$$|T|^2 = \frac{1}{3(2J_A+1)} \sum_{\mu_b \mu_d M_A M_B} \left| \sum_{\ell_c m_c \mu_c} \langle M_B | V | M_A, \ell_c, m_c, \mu_c \rangle \langle \ell_c, m_c, \mu_c, b | d \rangle \right|^2$$

The 3 and $(2J_A+1)$ are the number of magnetic substates of the deuteron and target A respectively.

Neglecting the d-state admixture in the deuteron wave function, one uses for χ_d the product of a triplet spin factor and an s-state (spherically symmetric) orbital factor:

$$\chi_{\mu_d}(\underline{r_b} - \underline{R}, \sigma_c, \sigma_b) = \psi_d(|\underline{r_b} - \underline{R}|) S_{\mu_d}(\sigma_c, \sigma_b).$$

Equation B4 then becomes $\langle \ell_c, m_c, \mu_c, b | d \rangle =$

$$\int Y_{\ell_c}^{m_c *}(\theta_c \phi_c) \chi_{\mu_c}^*(\sigma_c) \chi_{\mu_b}^*(\sigma_b) e^{-i\underline{k_b}\cdot\underline{r_b}} \psi_d(|\underline{r_b} - \underline{R}|) S_{\mu_d}(\sigma_c, \sigma_b) e^{-ik_d\cdot(\underline{R}+\underline{r_b})/2} d\underline{r_b}\, d\sigma_c\, d\sigma_b\, d\omega_c$$

$$= \langle \mu_c, \mu_b | \mu_d \rangle \, g_{\ell_c}^{m_c}$$

where

$$g_{\ell_c}^{m_c} = \int Y_{\ell_c}^{m_c *}(\theta_c \phi_c) e^{-i\underline{k_b}\cdot\underline{r_b}} \psi_d(|\underline{r_b} - \underline{R}|) e^{i\underline{k_d}\cdot(\underline{R}+\underline{r_b})/2} d\underline{r_b}\, d\omega_c \quad \underline{\text{B5}}$$

and

$$\langle \mu_c, \mu_b | \mu_d \rangle = \int \chi_{\mu_c}^*(\sigma_c) \chi_{\mu_b}^*(\sigma_b) S_{\mu_d}(\sigma_c \sigma_b)\, d\sigma_c\, d\sigma_b$$

is the Clebsch-Gordan coefficient corresponding to the composition $\frac{1}{2} \times \frac{1}{2}$ of the proton and neutron angular momenta with components μ_c, μ_b, into the deuteron triplet state with component μ_d. Hence

$$|T|^2 = \frac{1}{3(2J_A+1)} \sum_{\mu_b \mu_d M_A M_B} \left| \sum_{l_c m_c \mu_c} \langle M_B | V | M_A l_c m_c \mu_c \rangle \langle \mu_c \mu_b | \mu_d \rangle g_{l_c}^{m_c} \right|^2 .$$

The summations over μ_b, μ_d, and μ_c are considerably simplified by properties of the Clebsch-Gordan coefficients: $\langle \mu_c \mu_b | \mu_d \rangle = 0$ unless $\mu_c + \mu_b = \mu_d$. Therefore for a given term μ_b, μ_d in the outside summation, there is only one non-zero term in the coherent summation, which may thus be performed incoherently. Moreover one really sums over only two of the three quantum numbers μ_c, μ_b and μ_d, since they are so related. Expressing it as a sum over μ_c and μ_b leads to

$$|T|^2 = \frac{1}{3(2J_A+1)} \sum_{\substack{\mu_c \mu_b \\ M_A M_B}} \left| \langle \mu_c \mu_b | \mu_c + \mu_b \rangle \sum_{l_c m_c} \langle M_B | V | M_A l_c m_c \mu_c \rangle g_{l_c}^{m_c} \right|^2$$

Now all Clebsch-Gordan coefficients $\langle \mu_c \mu_b | \mu_d \rangle$ are zero except $\langle \frac{1}{2}, \frac{1}{2} | 1 \rangle = \langle -\frac{1}{2}, -\frac{1}{2} | -1 \rangle = 1$, and $\langle \frac{1}{2}, -\frac{1}{2} | 0 \rangle = \langle -\frac{1}{2}, \frac{1}{2} | 0 \rangle = \sqrt{\frac{1}{2}}$. Inserting these values gives

$$|T|^2 = \frac{1}{2(2J_A+1)} \sum_{M_A M_B \mu_c} \left| \sum_{l_c m_c} \langle M_B | V | M_A l_c m_c \mu_c \rangle g_{l_c}^{m_c} \right|^2$$

Following Bhatia et al., the summations over μ_c, M_A, M_B, can now be carried out with the aid of the group-theoretical properties of the wave functions concerned in the definition of

the Dirac bracket expressions $\langle M_B | V | M_A, \ell_c, m_c, \mu_c \rangle$. The result is

$$|T|^2 = \frac{1}{2(2J_A+1)} \sum_{\ell_c m_c} \frac{\Lambda_{\ell_c}}{2\ell_c+1} \left| g_{\ell_c}^{m_c} \right|^2 \qquad \underline{B6}$$

where

$$\Lambda_{\ell_c} = \sum_{\substack{m_c' \mu_c \\ M_A M_B}} \left| \langle M_B | V | M_A \ell_c m_c' \mu_c \rangle \right|^2$$

is usually kept as an adjustable parameter, since its honest evaluation would require more detailed knowledge of nuclear structure than is likely to be available for some time.

Finally $g_{\ell_c}^{m_c}$ can be evaluated from its definition, Equation B5. When performing the $\underline{r_b}$ integration, $\underline{r_c} = \underline{R} = (R, \theta_c, \phi_c)$ is constant, so that the integration over $\underline{r_b}$ can be replaced by an integration over the relative coordinate $\underline{\rho} = \underline{r_b} - \underline{R}$:

$$g_{\ell_c}^{m_c} = \int Y_{\ell_c}^{m_c *}(\theta_c \phi_c)\, e^{-i\underline{k_b}\cdot(\underline{\rho}+\underline{R})}\, \psi_d(\rho)\, e^{i\underline{k_d}\cdot(\underline{R}+\underline{\rho}+\underline{R})/2}\, d\underline{\rho}\, d\omega_c$$

$$= \int Y_{\ell_c}^{m_c *}(\theta_c \phi_c)\, e^{-i(\underline{k_b}-\frac{1}{2}\underline{k_d})\cdot\underline{\rho}}\, \psi_d(\rho)\, e^{i(\underline{k_d}-\underline{k_b})\cdot\underline{R}}\, d\underline{\rho}\, d\omega_c$$

$$= G(K_b) \int Y_{\ell_c}^{m_c *}(\theta_c \phi_c)\, e^{i\underline{k}\cdot\underline{R}}\, d\omega_c .$$

where $\underline{K_b} = \underline{k_b} - \frac{1}{2}\underline{k_d}$ represents that part of the outgoing nucleon's momentum which is obtained from its internal motion in the deuteron; whereas $\underline{k} = \underline{k_d} - \underline{k_b}$ (or more accurately $\underline{k} = \underline{k_d} - \frac{m_A}{m_B}\underline{k_b}$ when finite mass effects are considered) is the momentum with which the captured nucleon c approaches and

enters the target nucleus A (Figure 2). The symbol $G(\underline{K_b})$ denotes

$$G(\underline{K_b}) = \int e^{-i\underline{K_b}\cdot\underline{\rho}}\,\psi_d(\rho)\,d\underline{\rho} = \frac{4\pi}{K_b}\int_0^\infty \psi_d(\rho)\,\rho\,\sin(K_b\rho)\,d\rho = G(K_b) \qquad \underline{\text{B7}}$$

Finally, $\int Y_{\ell_c}^{m_c\,*}(\theta_c\,\phi_c)\,e^{i\underline{k}\cdot\underline{R}}\,d\omega_c$ is readily evaluated if the axis of quantization for determining ℓ_c is taken in the direction of $\underline{k}$. Starting with the well-known expansion of a plane wave into spherical harmonics,

$$e^{i\underline{k}\cdot\underline{R}} = \sum_{\ell_c'=0}^{\infty} i^{\ell_c'}\sqrt{4\pi(2\ell_c'+1)}\; j_{\ell_c'}(kR)\; Y_{\ell_c'}^{0}(\theta_c\,\phi_c), \qquad \underline{\text{B8}}$$

where

$$j_{\ell_c}(kR) = \left(\frac{\pi}{2kR}\right)^{1/2} J_{\ell_c+\frac{1}{2}}(kR)$$

is the spherical Bessel function, one obtains

$$\int Y_{\ell_c}^{m_c\,*}(\theta_c\,\phi_c)\,e^{i\underline{k}\cdot\underline{R}}\,d\omega_c = i^{\ell_c}\sqrt{4\pi(2\ell_c+1)}\; j_{\ell_c}(kR)\;\delta_{m_c,0}\,.$$

Hence,

$$\left|g_{\ell_c}^{m_c}\right|^2 \doteq G^2(K_b)\;4\pi\,(2\ell_c+1)\;j_{\ell_c}^2(kR)\;\delta_{m_c,0}$$

and substituting back into Equations B6 and B1,

$$|T|^2 = \frac{1}{2(2J_A+1)}\sum_{\ell_c\,m_c}\frac{\Lambda_{\ell_c}}{2\ell_c+1}\left|g_{\ell_c}^{m_c}\right|^2 = \frac{2\pi}{2J_A+1}\,G^2(K_b)\sum_{\ell_c}\Lambda_{\ell_c}\,j_{\ell_c}^2(kR)$$

and $\qquad \underline{\text{B9}}$

$$\sigma = \frac{m_b^*\,m_d^*}{4\pi^2\hbar^4}\,\frac{k_b}{k_d}\,|T|^2 = \frac{m_b^*\,m_d^*}{2\pi(2J_A+1)\hbar^4}\,\frac{k_b}{k_d}\,G^2(K_b)\sum_{\ell_c}\Lambda_{\ell_c}\,j_{\ell_c}^2(kR)$$

C- Discussion of the Cross-Section Formula

Equation B9, the final formula for the stripping cross-section, is conveniently expressed as

$$\sigma = \Pi(K_b) \sum_{l_c} L_{l_c}(k) \, P_{l_c} \qquad \text{C1}$$

where

$$\Pi(K_b) = (2\pi)^{-3} G^2(K_b) \qquad \text{C2}$$

$$L_{l_c}(k) = \sum_{m_c} \left| \int Y_{l_c}^{m_c *}(\theta_c \phi_c) e^{i\underline{k}\cdot\underline{R}} d\omega_c \right|^2 = 4\pi(2l_c+1) j_{l_c}^2(kR) \qquad \text{C3}$$

and

$$P_{l_c} = \frac{\pi m_b^* m_d^*}{\hbar^4} \frac{k_b}{k_d} \frac{\Lambda_{l_c}}{(2J_A+1)(2l_c+1)} . \qquad \text{C4}$$

These three factors have patent physical significance.

The first factor $\Pi(K_b)$ describes the break-up of the deuteron. It is the Fourier transform of the deuteron's internal wave function and thus gives the probability for the outgoing nucleon b to obtain from the deuteron's internal motion the momentum $\underline{K_b}$ which, added to half of the deuteron's external momentum $\underline{k_d}$, produces the observed outgoing momentum $\underline{k_b}$. (Figure 2). $\Pi(K_b)$ decreases smoothly and rapidly with increasing θ, and is not the factor mainly responsible for the typical peaked shape of the cross-section curve described qualitatively in Section A.

$L_{l_c}(k)$ is the centrifugal barrier factor for a nucleon of momentum k to reach the target boundary with angular momentum

ℓ_c, as discussed below. And P_{ℓ_c} is the probability that if such a nucleon does reach the target boundary it will be absorbed. P_{ℓ_c} is independent of k and of θ.

The characteristic peaks of the angular dependence are determined mainly by the centrifugal barrier factor $L_{\ell_c}(k)$. The significance of this factor is clear from the nature of the expansion of a plane wave into spherical harmonics used above (B8) to evaluate the integral $\int Y_{\ell_c}^{m_c *}(\theta_c \phi_c) e^{i\underline{k}\cdot\underline{R}} d\omega_c$. From general principles of quantum theory, the modulus squared of the coefficient $i^{\ell_c}\sqrt{4\pi(2\ell_c+1)}\, j_{\ell_c}(kR)$ of $Y_{\ell_c}^{0}(\theta_c \phi_c)$ in this expansion then gives the probability that the system described by the plane wave is in the state $Y_{\ell_c}^{0}(\theta_c \phi_c)$ at R. But this modulus squared is precisely

$$\left| i^{\ell_c}\sqrt{4\pi(2\ell_c+1)}\, j_{\ell_c}(kR)\right|^2 = 4\pi(2\ell_c+1)\, j_{\ell_c}^2(kR) = L_{\ell_c}(k) .$$

Thus the nucleon c may be pictured as issuing from the deuteron as a plane wave, which is equivalent to this sum of spherical waves of which only a few are effectively absorbed, since the selection rules make $P_{\ell_c} = 0$ for all but a few values of ℓ_c.

The properties of the nucleus exert their influence upon the probability of the reaction taking place, through these selection rules on the P_{ℓ_c}. The nucleus having thus declared which of the states $Y_{\ell_c}^{m_c}$ it is willing to accept and to what extent, the factor $L_{\ell_c}(k)$ determines how much of each such state is available in $e^{i\underline{k}\cdot\underline{R}}$, the plane wave representing the approach of the nucleon c. The factor $\Pi(K_b)$ represents the influence of the dynamics of the deuteron break-up in

determining the intensity of the plane wave $e^{i\underline{k}\cdot\underline{R}}$ as a function of the relative momentum k.

In a way it is rather surprising that such an interpretation should be a valid description, for the picture of a plane wave of nucleons c is even more virtual and subjective (in the sense of being a device for understanding, created by the physicist's mind) than most of the other constructs of physics. In the first place these three events — break-up of the deuteron, approach of c towards A in the form of a plane wave with momentum $\underline{k}$, and absorption of c by A — are separated only for convenience of description: they are not sequential in time. This makes the intermediate plane wave considerably less real than the compound nucleus in a Bohr-type nuclear reaction for instance, which, although not observed, must be postulated to exist for a specified length of time. Moreover the nucleon c must enter the nucleus with negative energy for a bound state to result, so that it is only by analogy that one can think of this plane wave as a solution of the free-particle Schrödinger equation, since the momentum k does not correspond to an energy $\frac{\hbar^2 k^2}{2m_c^*}$ in the usual way.

On the other hand it should be emphasized that the experimental evidence confirms this formula very clearly. The formula predicts a very special and recognizable angular dependence, governed by these $L_{\ell_c}(k)$ coefficients in the plane wave expansion, and the fact that the formula, admittedly approximate and far from unique, nevertheless fits the experimental data very well, is an encouragement to pursue this analogy in attempting to extend the theory. It should be borne

in mind that the angular distributions are determined primarily by the momentum balance, and the momentum balance is preserved artfully in this perturbation approach. Neutron-gamma angular correlations discussed in the next section also support this picture.

It is interesting that the Butler formula obtained by smooth fitting of wave functions at the nuclear boundary, can also be cast into the form C1,

$$\sigma = \Pi(K_b) \sum_{\ell_c} L_{\ell_c}(k)\, P_{\ell_c}$$

and so shares in the above interpretation to a certain extent, if less directly. The factor $\Pi(K_b)$ is unchanged. Instead of involving simply a spherical Bessel function $j_{\ell_c}(kR)$, the factor $L_{\ell_c}(k)$ in the Butler formula involves the Wronskian of $j_{\ell_c}(kR)$ with a function independent of angle, so that it contains also a correction term proportional to the derivative of $j_{\ell_c}(kR)$. This correction term, which is in fact dominated by the first term in all cases where the basic stripping approximations are justified, can be neglected in most practical applications, and its effect simulated by using a radius artificially larger by about one fermi in the Born approximation formula than in the Butler formula. The effective radius in both theories appears more as an adjustable parameter than as a physically meaningful quantity.

The Butler derivation gives for P_{ℓ_c}, in the case of a bound final state,

$$P_{\ell_c} = \frac{\pi R^2}{2} \frac{m_d^* m_b^* k_b}{m_c^* k_d} \frac{(2J_B + 1)}{(2J_A + 1)2(2\ell_c + 1)} F_{\ell_c} \qquad \underline{C5}$$

where F_{ℓ_c} is the probability density that the state of B can yield c at the surface with the orbital angular momentum ℓ_c, leaving A in its ground state. More precisely, let the normalized wave function $\chi_{M_B}(\underline{r_c}, \sigma_c, \xi)$ in the region $r_c > R$ (where B breaks up into the non-interacting constituents A and c) be expanded as

$$\chi_{M_B}(\underline{r_c}, \sigma_c, \xi) \sim \sum u_{t,M_A}(\xi)\, Y_{\ell_c}^{m_c}(\theta_c \phi_c)\, \chi_{\mu_c}(\sigma_c)\, \Psi^{t,\ell_c}_{M_A m_c \mu_c; M_B}(r_c),$$

where the functions $u_{t,M_A}(\xi)$ describe the states of A (with angular momentum J_A, M_A), and $\Psi^{t,\ell_c}_{M_A m_c \mu_c; M_B}(r_c)$ is a free-particle radial wave function for c. Then F_{ℓ_c} is given by

$$F_{\ell_c} = \sum_{M_A, m_c, \mu_c} \left| \Psi^{t,\ell_c}_{M_A m_c \mu_c; M_B}(R) \right|^2$$

where t = 0 designates the ground state of A.

On the other hand, the Butler result for the case of a <u>virtual</u> (instead of bound) level of B is given by

$$P_{\ell_c} = \frac{\pi}{\hbar^2} \frac{m_d^* m_b^*}{m_c^*} \frac{k_b}{k_d} \frac{(2J_B+1)}{(2J_A+1)2(2\ell_c+1)} \gamma_{\ell_c}^2 . \qquad \underline{C6}$$

Here $\gamma_{\ell_c}^2$ is the reduced width of the level, related to its partial width Γ_{ℓ_c} (for decay into c and the ground state of A, with angular momentum ℓ_c) by

$$\gamma_{\ell_c}^2 = \tfrac{1}{2} k_c R^2 \left| h_{\ell_c}^{(1)}(k_c R) \right|^2 \Gamma_{\ell_c}$$

One can, by analogy, formally define a "reduced width" for the <u>bound</u> levels also,

$$\gamma_{\ell_c}^2 = \frac{\hbar^2 R^2}{2m_c^*} F_{\ell_c}$$

which makes Equation C6 coincide with C5. ($h^{(1)}_{\ell_c}$ is a spherical Hankel function of the first kind, and k_c is such that $\frac{\hbar^2 k_c^2}{2m_c^*}$ is the energy of the virtual level; k_c is not the same as k, and does not depend on θ.) The Born approximation formula C4 for P_{ℓ_c} can be reconciled with the Butler form C5 by formal manipulations.

To close this discussion let us note from Equation B9 that the contributions of the various values of ℓ_c allowed by conservation of angular momentum, i.e.

$$|J_A - J_B| - \tfrac{1}{2} \leq \ell_c \leq J_A + J_B + \tfrac{1}{2} \qquad \underline{C7}$$

are to be summed incoherently, and that in practice all but one of these values can be neglected; the one important value is usually the smallest allowed, except in some interesting cases where this is excluded by nuclear properties. These cases are usually explicable in terms of the shell model.

D- Angular Correlations

The theory has been extended, by Satchler (5) and by others, to describe the angular correlations between the direction of emission of nucleon b and that of the subsequent gamma ray of de-excitation, in cases where B is left in an excited state. An angular correlation function gives simply a coincidence counting rate as a function of the two directions: how many gamma rays are recorded in one direction in

coincidence with particles b in another direction.

It is an interesting confirmation of the picture presented in the previous section that the angular correlations can be accounted for on the following basis. Particle c is pictured as entering nucleus A as a plane wave of momentum k, which momentum is related to the direction of emission of b; the subsequent angular distribution of gamma rays with respect to the direction k is calculated exactly on the same basis as if one were studying the reaction $c + A \rightarrow (B^*) \rightarrow B + \gamma$ by means of bombardment of A by a beam of particles c incident in the direction k.

It follows that the angular correlation curves should be symmetrical with respect to this direction k, which is the recoil direction. This general property of stripping angular correlations is usually confirmed by experiment whenever the angular distributions show characteristic stripping patterns.

The distribution of gamma ray directions following such bombardment is given quite simply, in cases of pure multipolarity and short enough lifetime ($\tau \leq 10^{-11}$ sec.) to avoid complications, in terms of functions which will now be defined. This outline follows mainly an unpublished report by Edwards (6) which gives in less up-to-date but somewhat more explicit form an account of his published work (7).

The pure multipole fields are a complete set of solutions of the free-space Maxwell equations, characterized by their parity properties. Instead of going through the general solution of this problem involving the vector spherical

harmonics, we need to consider only a more restricted and easily obtained set of functions capable of describing the simpler fields prevailing in the so-called "far field" or "wave zone" region, i.e. far from the nucleus which is the source of the field. In that region, the fields $\underline{\mathcal{E}}$ and $\underline{\mathcal{H}}$ are almost transverse and the propagation vector, or the vector $\underline{G}$ giving the linear momentum density of the field,

$$\underline{G} = \frac{\underline{\mathcal{E}} \times \underline{\mathcal{H}}}{4\pi c}$$

which is proportional to it, is almost parallel to the radius vector $\underline{r}$.

We can not, however, make the simplifying assumption that $\underline{\mathcal{E}}$ and $\underline{\mathcal{H}}$ are both perpendicular to $\underline{r}$, because we are specifically interested in describing the removal of a certain quantity of angular momentum through emission of a photon, and the angular momentum density of the field

$$\underline{\mathcal{L}} = \frac{\underline{r} \times \underline{G}}{4\pi c}$$

of course vanishes under these oversimplified conditions.

On the other hand we can assume that only one of the two vectors $\underline{\mathcal{E}}$ and $\underline{\mathcal{H}}$ has a non-vanishing component in the $\underline{r}$ direction, while the other is always perpendicular to $\underline{r}$. Then since $\underline{\mathcal{E}}$ and $\underline{\mathcal{H}}$ are perpendicular to each other and equal in magnitude in the far field region, the angular dependence is given by the magnitude

$$|\underline{S}| = \frac{c}{4\pi} \mathcal{E}^2 = \frac{c}{4\pi} \mathcal{H}^2 \qquad \underline{\text{D1}}$$

of the Poynting vector

$$\underline{S} = \frac{c}{4\pi}\, \underline{\mathcal{E}} \times \underline{\mathcal{H}}$$

We can substitute into D1 an expression for either $\mathcal{E}$ or $\mathcal{H}$, whichever is the simpler.

Let us construct then a complete set of vector point functions of θ and ϕ, everywhere perpendicular to $\underline{r}$. Taking the gradient

$$\underline{\nabla} = \hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial\theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}$$

of the scalar (ordinary) spherical harmonics $Y_{LM}(\theta,\phi)$ is the simplest way to obtain a set of vector point functions of θ and ϕ; cross-multiplying by $\underline{r}$ eliminates the r-dependence introduced by the gradient operation, and makes the functions always perpendicular to $\underline{r}$:

$$\underline{r} \times \underline{\nabla} = \hat{\phi}\frac{\partial}{\partial\theta} - \hat{\theta}\frac{1}{\sin\theta}\frac{\partial}{\partial\phi} .$$

For convenience the set of functions $\underline{X}_{LM}(\theta,\phi)$ is defined as

$$\underline{X}_{LM}(\theta,\phi) = \frac{-i\,\underline{r}\times\underline{\nabla}\, Y_{LM}(\theta\phi)}{\sqrt{L(L+1)}} = \frac{-i}{\sqrt{L(L+1)}}\left\{\hat{\phi}\frac{\partial}{\partial\theta} - \hat{\theta}\frac{1}{\sin\theta}\frac{\partial}{\partial\phi}\right\} Y_{LM}(\theta\phi)$$

Because the operator $\underline{r} \times \underline{\nabla}$ commutes with the angular momentum operator of which the Y_{LM} are a complete orthogonal set of eigenfunctions, these $\underline{X}_{LM}$ are a complete orthogonal set of functions perpendicular to $\underline{r}$.

To discuss the case where the vector $\underline{\mathcal{H}}$ is perpendicular to $\underline{r}$ in the far field region, we define

$$\underline{\mathcal{H}}^{\varepsilon}_{LM}(\underline{r}) = g^{\varepsilon}_{LM}(r)\, \underline{X}_{LM}(\theta\phi) .$$

It then follows from the free-space Maxwell equation that the corresponding $\underline{\mathcal{E}}$ is

$$\underline{\mathcal{E}}^{\epsilon}_{LM}(\underline{r}) = \frac{ic}{\omega}\,\underline{\nabla} \times g^{\epsilon}_{LM}(r)\,\underline{X}_{LM}(\theta,\phi).$$

Now the coefficients $g^{\epsilon}_{LM}(r)$ can be suitably chosen so that the $\underline{\mathcal{H}}$ satisfies the free-space Maxwell equation. Moreover, since the $Y_{LM}(\theta,\phi)$ have parity $(-1)^L$, it follows that $\mathcal{H}^{\epsilon}_{LM}(\underline{r})$ as defined has parity $(-1)^L$ and $\mathcal{E}^{\epsilon}_{LM}(\underline{r})$ has parity $-(-1)^L$. The statements of this paragraph amount to a definition of what is known as an <u>electric</u> multipole field of multipolarity (L,M).

On the other hand we could have started by assuming that the vector $\underline{\mathcal{E}}$ instead of $\underline{\mathcal{H}}$ was perpendicular to $\underline{r}$ in the far field region, and constructed a vector point function

$$\underline{\mathcal{E}}^{m}_{LM}(\underline{r}) = g^{m}_{LM}(r)\,\underline{X}_{LM}(\theta,\phi)$$

with the $g^{m}_{LM}(r)$ suitably chosen to satisfy the Maxwell equations for $\underline{\mathcal{E}}$. This $\underline{\mathcal{E}}^{m}_{LM}(\underline{r})$ has parity $(-1)^L$ and is accompanied by an $\underline{\mathcal{H}}^{m}_{LM}(\underline{r})$ having parity $-(-1)^L$. This set of $\underline{\mathcal{E}}^{m}_{LM}(\underline{r})$ and $\underline{\mathcal{H}}^{m}_{LM}(\underline{r})$ then corresponds to the definition of what is known as the pure <u>magnetic</u> multipole field of multipolarity (L,M).

Having obtained such expressions for the electric and magnetic multipole fields, it is a simple matter to obtain the angular distribution of the corresponding gamma rays, through that of the Poynting vector giving the energy flow. For the electric multipole field, one finds

$$|\underline{S}^{\epsilon}_{LM}| = \frac{c}{4\pi}\,\mathcal{H}^{\epsilon\,2}_{LM} = \frac{c}{4\pi}\,|g^{\epsilon}_{LM}(r)|^2\,\underline{X}^{*}_{LM}(\theta,\phi)\cdot\underline{X}_{LM}(\theta\phi)$$

and similarly for the magnetic multipole fields:

$$|\underline{S}^{m}_{LM}| = \frac{c}{4\pi}\,\mathcal{E}^{m\,2}_{LM} = \frac{c}{4\pi}\,|g^{m}_{LM}(r)|^{2}\,\underline{X}^{*}_{LM}(\theta,\phi)\cdot\underline{X}_{LM}(\theta,\phi).$$

Thus in both cases the angular dependence is contained in the functions

$$F_{L|M|}(\theta) = \underline{X}^{*}_{LM}(\theta,\phi)\cdot\underline{X}_{LM}(\theta,\phi)$$

$$= \frac{1}{L(L+1)}\left\{\left|\frac{\partial}{\partial\theta}Y_{LM}(\theta,\phi)\right|^{2} + \left|\frac{1}{\sin\theta}\frac{\partial}{\partial\phi}Y_{LM}(\theta,\phi)\right|^{2}\right\}$$

which are called the classical multipole distribution functions. Tables of these are available (Arnold, 8).

In applying this theory to the stripping reaction leading to the first excited state of B, with spin J_e, which subsequently decays to the ground state with spin J_f (subscripts e for excited, f for final state) by a one-step pure multipole transition† of multipolarity (L,M), it will be necessary, however, to perform coherent summations over the possible values of any unobserved intermediate quantum numbers, namely the various possible combinations of magnetic substates. Hence one must use the appropriate amplitude expressions $\underline{X}_{LM}(\theta,\phi)$ instead of the probability expressions $F_{L|M|}(\theta)$ in order to obtain the coherent effects on the angular distributions.

In particular if the system is in a magnetic substate M_e of J_e, the amplitude function corresponding to its decay to a particular substate M_f of J_f is given by $\underline{X}_{LM}(\theta,\phi)$, with $M = M_e - M_f$, and this is to be summed over the various possible M_f with weighting factors giving the relative probability

† One can show (46) that the multipole fields as defined here do carry away angular momentum (L,M).

amplitude of the transitions M_e to M_f.

Let $\Gamma_e(R)$, $\Gamma_f(R)$ represent the radial functions of the internal coordinates of B corresponding to excited and final states respectively, $\xi_e(J_e, M_e)$, $\xi_f(J_f, M_f)$ represent the corresponding total angular momentum eigenfunctions, and $\Psi(L,M)$ the wave function for the photon emitted. The internal wavefunction of the excited state can then be written $\Gamma_e(R)\ \xi_e(J_e, M_e)$ and that of the final state $\Gamma_f(R)\ \xi_f(J_f, M_f)\Psi(L,M)$. The angular part of this final state wave function is conveniently expanded in terms of excited state angular functions:

$$\xi_f(J_f, M_f)\,\Psi(L,M) = \sum_{J_e' M_e'} C^{J_e' J_f L}_{M_e' M_f M}\ \xi_e(J_e', M_e')$$

where $C^{J_e' J_f L}_{M_e' M_f M}$ is the Clebsch-Gordan coefficient corresponding to $\underline{J_e'} = \underline{J_f} + \underline{L}$ with $M_e' = M_f + M$. The relative probability amplitudes of the transitions $M_e \rightarrow M_f$ are then

$$\left\langle \sum_{J_e' M_e'} C^{J_e' J_f L}_{M_e' M_f M}\ \xi_e(J_e' M_e') \,\middle|\, \xi_e(J_e M_e) \right\rangle = C^{J_e J_f L}_{M_e M_f M}$$

and the amplitude for the angular distribution resulting from the transition $M_e \rightarrow M_f$ becomes proportional to

$$A_\gamma(M_e, \theta, \phi) = C^{J_e J_f L}_{M_e M_f M}\ \underline{Y}_{LM}(\theta\phi)$$

This amplitude must be multiplied by a further weighting factor, namely the probability amplitude that the stripping process will leave B^* in substate M_e, and then summed

coherently over the possible values of M_e. This stripping amplitude can be evaluated for instance from Equation B3, with $M_\beta = M_e$, except that one normally neglects the sum over l_c, using only the one value of l_c which is known from the angular distribution to be predominant; measurements are usually taken with the b-counter at the major peak of the angular distribution corresponding to that l_c, so that this procedure should be justified. Final expressions must as usual be summed over possible values of the final and averaged over those of the initial unobserved quantum numbers, so that the angular correlation function is proportional to

$$\frac{1}{3(2J_A+1)} \sum_{M_A M_d M_f M_c} \left| \sum_{M_e} T_{M_e}(\theta_b) A_\gamma(M_e, \theta_\gamma, \phi_\gamma) \right|^2 \qquad \underline{\text{D2}}$$

where $T_{M_e}(\theta_b)$ denotes the stripping amplitude or matrix element, as in Equation B2 with $M_\beta = M_e$. ($\theta_\gamma, \phi_\gamma$ refer to the gamma-ray direction and θ_b, written θ in other sections, to that of b.)

<u>E- The Reaction B^{11} (d,n)C^{12*} 4.43 Mev</u>

The basic stripping theory presented in the preceding sections has proved extremely successful and useful in accounting for experimental angular distributions of many (d,n) and (d,p) reactions, not only under conditions where its basic assumptions are justified, but even in many cases where they are not. There are several reactions however for which the experimental data do not exhibit the usual characteristic

stripping pattern. Because of the success obtained in other cases, attempts have been made to account for the anomalous results by modifications and extensions within the general framework of deuteron stripping theory.

One of these reactions, $B^{11}(d,n)C^{12*}$ leading to the first excited state of C^{12} at 4.43 Mev, forms a major subject of this thesis. The experimental data are in the range of bombarding energies from $\frac{1}{2}$ to 2 Mev.

The B^{11} ground state has spin/parity $3/2^-$, as expected on the basis of the shell model. The ground state of C^{12} has 0^+, and the 4.43 Mev first excited state 2^+, as is normal for even-even nuclei. (The gamma ray of de-excitation is pure quadrupole as it should be.) Conservation of total angular momentum in the formation of the 2^+ state thus restricts l_c to $0 \leq l_c \leq 4$ according to Equation C7; and conservation of parity further restricts l_c to the odd values $l_c = 1$ and 3.

From pure stripping theory, the angular distribution expected (C1) is therefore proportional to $\pi(K_b)\ j_1^2(kR)$ plus some minor contribution of $\pi(K_b)\ j_3^2(kR)$. For $\frac{1}{2}$ Mev, say, the distribution $\sigma(\theta)$ should start from a minimum (seldom actually seen) at $\theta = 0$, rise to a peak around 10° to 15°, decrease sharply to a zero around 105°, and then rise slightly towards a flat secondary maximum around 175°, this secondary maximum being only about 6% as high as the forward maximum.

The experimental curves actually obtained, e.g. by Neilson et al (9), are very different. They show a maximum at $\theta = 0$, decrease only slightly to a minimum whose position varies from

about 65° at $\frac{1}{2}$ Mev to about 90° at 2 Mev; then rise again for larger angles. The ordinate at the minimum is from 70% to 50% of the height of the forward peak, instead of going down to "zero", and the backward maximum is from 80% to 60% as high as the forward at the highest angles reached, about 145°.

The experimental angular distribution for a bombarding energy of $\frac{1}{2}$ Mev is presented in Figure 4, along with a stripping curve for these conditions. The data at the other bombarding energies show similar disagreement with theory. It will be more convenient to present them for detailed study in a different form (plotted as a function of k instead of θ) in Chapter 3.

Neutron-gamma angular correlations for this reaction have also been measured by Neilson et al (9). They do not show symmetry about the recoil direction $\underline{k}$, as they should on the basis of pure stripping theory (see Section D).

A reasonable radius for the nucleus B^{11} is of the order of 3.5 fermis, which implies that the deuteron, or at least its proton constituent, must approach to a radius of about 4.5 fermis if it is to be stripped off by the attractive nuclear forces. The height of the Coulomb barrier as seen by the proton at this radius would be of the order of 1.6 Mev even for $\ell_c = 0$. For $\ell_c = 1$ the centrifugal barrier increases the barrier height to 4.1 Mev, at a distance of 4.5 fermis. Hence it would not be unreasonable to expect the stripping pattern to be very considerably modified by Coulomb effects at the energies considered. On the other hand, there are other reactions where a recognizable stripping pattern persists even under such

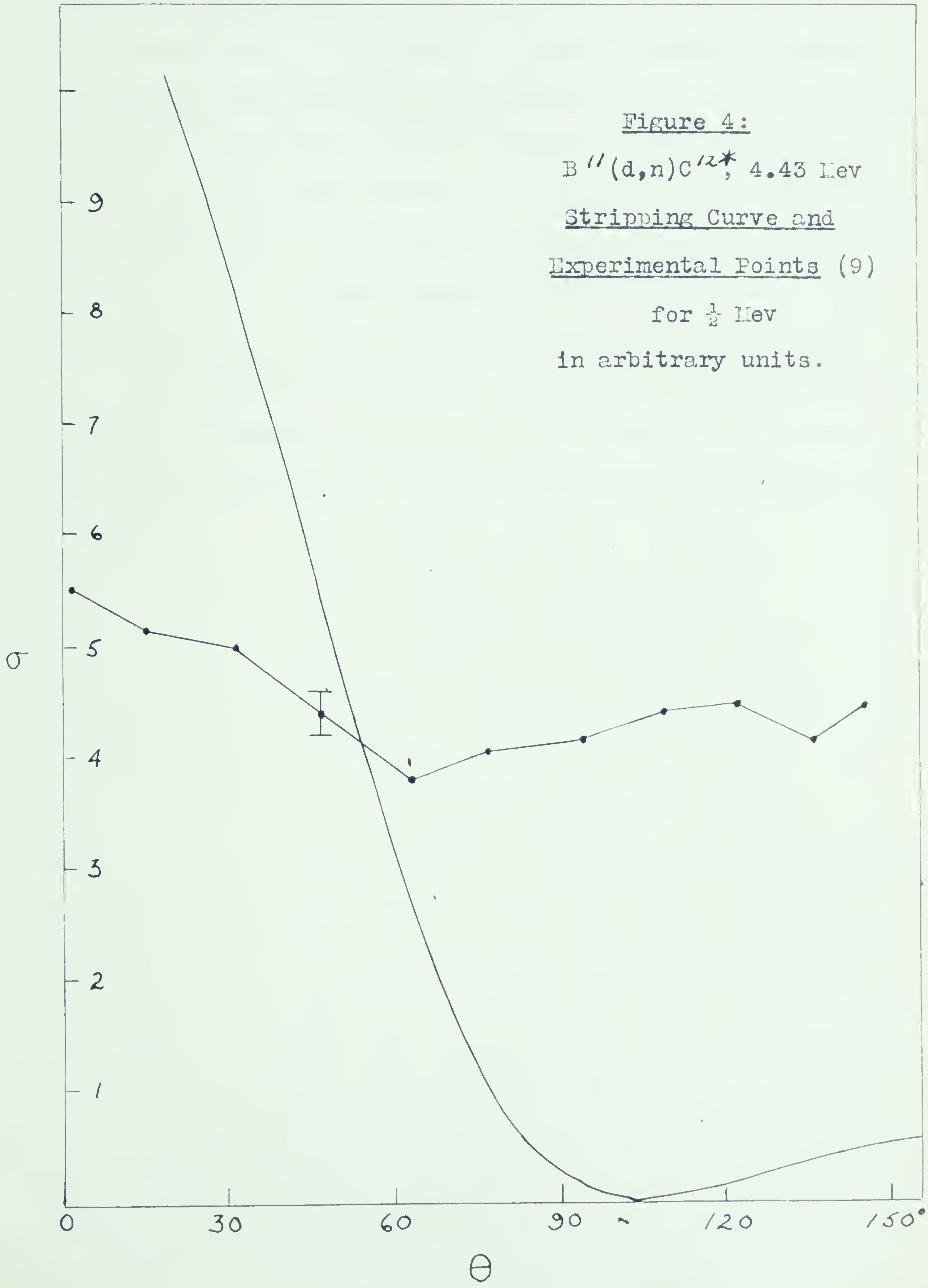


Figure 4:

$B^{11}(d,n)C^{12*}$, 4.43 Mev

Stripping Curve and Experimental Points (9)

for ½ Mev

in arbitrary units.

adverse conditions.

Recent work by Wilkinson (10) has suggested reasons why stripping reactions with low Q-values, say from -2 to + 2 Mev, might be expected to exhibit persistent stripping patterns even at low energies. The reaction $B^{11}(d,n)C^{12*}$ has a high Q-value of 9.30 for this level and so is not in that category; on the contrary the Wilkinson argument would predict a much less characteristic stripping pattern for such a high Q-value.

The explanation of these anomalous distributions by heavy-particle stripping will be critically reviewed in Chapter 2, and the possibility of other explanations investigated in Chapter 3.

Chapter 2 HEAVY-PARTICLE STRIPPING

F- Description of Heavy-Particle Stripping

The unexpected angular distributions obtained experimentally for $B^{11}(d,n)C^{12*}$(4.43 Mev level) have been described in Section E above. These and similar results obtained in a few other reactions have led a group at Johns Hopkins University to postulate an effect called heavy-particle stripping. They have published several papers in connection with this proposal (11 to 19, 7). This hypothesis will be described qualitatively in this section, and examined more critically in the remainder of this chapter.

The essential idea of the ordinary deuteron stripping mechanism in a (d,n) reaction is that the outgoing neutron comes from the deuteron, not the target (Figure 1). This neutron bears no label of course, but its identity is established by the success of deuteron stripping theory in predicting such data as angular distributions. But suppose now the B^{11} target to consist of a compact B^{10} core with the extra neutron loosely bound to it. When this system collides with the deuteron, it is quite conceivable that the B^{10} core and deuteron should merge, forming a C^{12*} residual nucleus, while the loose outer neutron from B^{11}, only distantly involved in the collision, is simply shaken off and leaves the scene of action with the momentum it had at that instant (Figure 3).

This mode of reaction is called heavy-particle stripping, by analogy with deuteron stripping: in this case it is the heavy particle, (i.e. the B^{11}), rather than the deuteron, which is broken up into two parts. One of these parts, the B^{10} core, hits and amalgamates with the other party to the collision, namely the deuteron; the other part is the outgoing neutron. For comparison, in the deuteron stripping mode, it is the deuteron which is broken up into two parts: one of these parts, the proton, hits and amalgamates with the other party to the collision, namely the B^{11}; the other part is the outgoing neutron.

The idea therefore is to describe heavy-particle stripping by the usual deuteron stripping theory, simply replacing the deuteron by the B^{11} system, its proton by the postulated B^{10} core of the B^{11}, and the B^{11} target by the deuteron.

The qualitative reason why this heavy-particle stripping mode is suspected of being responsible for the anomalous angular distributions is simply this: if the outgoing neutron comes from the B^{11} target, and its final momentum is compounded of its share of the momentum of the whole target plus some contribution from its internal motion in the B^{11}, backward momenta ($\theta \simeq 180^\circ$) should be strongly favored, because the target's momentum in the center-of-mass system before collision is in the direction $\theta = 180^\circ$. Just as deuteron stripping implied a strong forward peak in the neutron angular distribution because the neutron was originally part of the forward-moving deuteron, heavy-particle stripping implies a strong backward

peak in the neutron angular distribution because the neutron was originally part of the backward-moving heavy-particle B''. Now since both a forward and a backward peak occur in the anomalous experimental results, it has been concluded (16) that they may be due to the simultaneous occurrence of both modes of reaction.

The idea of reactions proceeding by more than one mode is of course not special to this reaction. Because of the fundamental indistinguishability of elementary particles in quantum mechanics, it is never certain, in atomic or nuclear reactions involving more than one particle of the same species, whether the observed reaction product was originally part of the projectile or of the target. The standard procedure to take this into account is to antisymmetrize all wave functions with respect to the coordinates of the identical particles. Observable quantities calculated from such wave functions, (e.g. matrix elements or reaction cross-sections), will therefore contain both direct and exchange terms. The direct terms are identical with the answer that would have been obtained if the wave functions had not been antisymmetrized.

The reason why the antisymmetrization can often be neglected in practice is that the exchange terms turn out to be small. This will be true in all cases where the observed reaction product "obviously" has a much greater probability of coming from one of the reactants than from the other. For example, ordinary deuteron stripping depends on the fact that the deuteron is a very loosely bound structure as compared to

other nuclei. Antisymmetrization is neglected in ordinary stripping theory because nucleons within the target nuclei are normally much more tightly bound than in the deuteron, and therefore less subject to the type of collision leading to stripping. Mathematically this is expressed by small overlap between the neutron wave function in the nucleus and that in the deuteron, so that exchange integrals are small.

Thus heavy-particle stripping is always present in principle but normally negligible in magnitude compared to deuteron stripping. The question is whether there is reason to believe that in this particular case it happens to be comparable in importance with deuteron stripping.

The mathematical manipulations whereby the two stripping modes are considered in the evaluation of cross-sections will be reviewed in the next section. This will be followed by sections dealing with the question of relative magnitudes, and with other weak points in the two-mode theory.

G- Formal Combination of the two Modes

The evaluation of the stripping matrix element in two-mode stripping follows the particular version of stripping theory which has been presented in Chapter 1. The important new element, the antisymmetrization, is introduced as follows.

In the spirit of the Born approximation, matrix elements of the form

$$\langle \psi_{final} | V | \psi_{total} \rangle \approx \langle \psi_{final} | V | \psi_{initial} \rangle$$

will be considered. One of the two wave functions, initial or final, must be antisymmetrized. (It can be shown that it is not necessary to antisymmetrize both wave functions if the interaction potential is symmetrical with respect to the identical particles.)

B^{11} can be assumed to contain two neutrons deeply bound in the 1s shell, and four outer neutrons in a p 3/2 level. These four outer neutrons are the only ones considered, on the grounds that the s neutrons are much less likely to become separated from the nucleus in a stripping type of collision. The initial wave function is therefore antisymmetrized with respect to the interchange of any pair of the five neutrons which may turn out to be the outgoing one, namely the one originally in the deuteron, and the four originally in the p 3/2 outer shell of the B^{11} nucleus.

The various terms in the wave function representing the initial state will contain a factor representing the neutron from the deuteron, and a factor representing the four neutrons in the p-shell of the B^{11}. Since only the neutrons will enter the discussion for now, these terms can be abbreviated like this:

$$| \, i \ 1,2345 \rangle$$

The i means initial, the labelling number written before the comma denotes the neutron in the deuteron, and the four labelling numbers written after the comma denote the neutrons in the p-shell of the target.

Now the wave function describing four neutrons in this shell must itself be antisymmetric with respect to interchange of any pair of the four neutrons, and since it appears as a factor in the various terms, this property greatly simplifies the antisymmetrization that must be performed with respect to the five neutrons. In fact an antisymmetric function can be written in just five terms:

$$\left| \overset{A}{i}\ 12345 \right\rangle \propto \left| i\ 1,2345 \right\rangle - \left| i\ 2,1345 \right\rangle - \left| i\ 3,2145 \right\rangle - \left| i\ 4,2315 \right\rangle - \left| i\ 5,2341 \right\rangle$$

as can be verified by taking the various permutations on two particles.

The final state wave function, which need not be treated similarly, can be written $\left| f\ 1,2345 \right\rangle$, where f stands for final, the first digit represents the outgoing neutron, and the four digits after the comma represent the four equivalent neutrons left in the p-shell of C^{12*}. It is of course antisymmetric under interchange of any pair of these four indices after the comma.

The interaction potential V would in general contain several terms, describing interactions between neutron and target, proton and target, and neutron and proton, in the case of deuteron stripping; or neutron and deuteron, core and

deuteron, neutron and core, in the case of heavy-particle stripping. However, (as in Section B) the stripping approximations consist essentially in the neglect of all interactions but the one between the two objects which actually hit and merge, namely V_{pT} for deuteron stripping where the proton hits and is absorbed by the B^{11} nucleus, or V_{dC} for heavy-particle stripping where the B^{10} core from B^{11} hits and coalesces with the deuteron (subscripts p for proton, T for the B^{11} target, d for deuteron, and C for the B^{10} core of B^{11}).

Thus the matrix elements of these interactions are to be evaluated between a five-term wave function describing the initial state and a single term wave function describing the final state, in order to give a transition probability amplitude T. In the Johns Hopkins publications the simplification of the matrix element by means of the stripping approximations is a highly formal procedure, involving expansion of the five-term matrix element in terms of a set of initial state Green's functions, and neglect of all but the leading terms in accordance with the stripping approximations relevant in each case. We can see in a more physical way the significance of these manipulations. We have

$$T \propto \langle f\ 1,2345 | V | i\ 1,2345 \rangle - \langle f\ 1,2345 | V | i\ 2,1345 \rangle - \langle f\ 1,2345 | V | i\ 3,2145 \rangle - \langle f\ 1,2345 | V | i\ 4,2315 \rangle - \langle f\ 1,2345 | V | i\ 5,2341 \rangle .$$

Now, remembering the meaning of the comma notation, we interpret the first term $\langle f\ 1,2345 | V | i\ 1,2345 \rangle$: here the

neutron which was originally in the incident deuteron is the one labelled 1, and is the one that goes out and is observed: ordinary deuteron stripping has taken place; hence, according to the stripping approximations, let us keep only V_{pT} in this case.

But in the second term $\langle f\ 1,2345 \mid V \mid i\ 2,1345 \rangle$, the neutron originally in the incident deuteron is the one labelled 2, and it is not the same as the outgoing neutron: it goes into the C^{12*} while the neutron labelled 1, originally in the B^{11}, becomes the outgoing particle. Hence we say that heavy-particle stripping has taken place and for this potential V we keep only $V_{d(2)C}$, the interaction between the deuteron (containing the neutron labelled 2) and the B^{10} core.

Similarly the three other terms $\langle f\ 1,2345 \mid V \mid i\ 3,2145 \rangle$, $\langle f\ 1,2345 \mid V \mid i\ 4,2315 \rangle$ and $\langle f\ 1,2345 \mid V \mid i\ 5,2341 \rangle$ correspond to neutrons 3, 4, and 5, respectively, in the incoming deuteron, while neutron 1, originally part of the target, is always the outgoing one. This means heavy-particle stripping; hence, by the stripping approximations, we drop all V terms but $V_{d(3)C}$, $V_{d(4)C}$ or $V_{d(5)C}$ respectively.

Now each such term in the matrix element is just a number, the result of a multiple integration and/or summation: the labels 1, 2, ... denote different variables of integration. One can therefore relabel at will. The exchange terms are now relabelled so that in each case the neutron from the deuteron gets the label 1, and the outgoing neutron gets the label 2. To this end the operator $\binom{12345}{21345}$ interchanges the labels 1 and 2

in the first exchange term $\langle f\ 1,2345 \mid V_{d(2)C} \mid i\ 2,1345 \rangle$, giving $\langle f\ 2,1345 \mid V_{d(1)C} \mid i\ 1,2345 \rangle$. Similarly,

$$\binom{12345}{23145} \langle f\ 1,2345 \mid V_{d(3)C} \mid i\ 3,2145 \rangle \rightarrow \langle f\ 2,3145 \mid V_{d(1)C} \mid i\ 1,3245 \rangle$$

$$\binom{12345}{24315} \langle f\ 1,2345 \mid V_{d(4)C} \mid i\ 4,2315 \rangle \rightarrow \langle f\ 2,4315 \mid V_{d(1)C} \mid i\ 1,4325 \rangle$$

$$\binom{12345}{25341} \langle f\ 1,2345 \mid V_{d(5)C} \mid i\ 5,2341 \rangle \rightarrow \langle f\ 2,5341 \mid V_{d(1)C} \mid i\ 1,5342 \rangle$$

Each of the last three terms in the form obtained above can now be shown to be equal to the term $\langle f\ 2,1345 \mid V_{d(1)C} \mid i\ 1,2345 \rangle$ by means of interchanges of labels within the p-shell wave functions only, using of course the antisymmetry property of these wave functions. It can readily be verified that whenever an odd number of interchanges is necessary to bring the initial wave function to the form $\mid i\ 1,2345 \rangle$, thus introducing a minus sign, another minus sign cancelling the first one will similarly be produced by changing the final wave function to the form $\mid f\ 2,1345 \rangle$.

In this way the complete matrix element for two-mode stripping reduces to the interesting form

$$T \propto \langle f\ 1,2345 \mid V_{pT} \mid i\ 1,2345 \rangle - 4 \langle f\ 2,1345 \mid V_{d(1)C} \mid i\ 1,2345 \rangle \tag{G1}$$

This completes the process of mixing in the contributions of the two modes. The first term is given by pure stripping theory as in Chapter 1, and the evaluation of the second, the exchange or heavy-particle stripping term, follows that theory as closely as possible, with appropriate changes of course.

If one is interested in the neutron angular distribution, the cross-section (B1) is obtained essentially by squaring this matrix element T, so that it will contain a deuteron stripping term, a heavy-particle stripping term, and interference terms. The angular correlation between the neutron and gamma directions is obtained by inserting the matrix element T of G1 into D2.

H- Evaluation of the matrix elements

In two-mode stripping theory, the evaluation of the matrix element for pure deuteron stripping, $\langle f\ 1,2345 \mid V_{\rho\tau} \mid i\ 1,2345 \rangle$ is performed by the method of Chapter 1 but presented differently, and further assumptions and approximations are made. Since the physical ideas are the same and Edwards' text (6,7) is clear on this point, it will not be necessary to introduce the complicated notational system found to be necessary to carry this out explicitly.

The procedure is to carry out expansions of the various factors or groups of factors of the initial state wave function in terms of corresponding factors of the final state wave function, using the physical nature of the process as a guide in choosing the proper expansions. Most of the summations introduced reduce to a single term because of the experimental conditions (e.g. selecting only states with a given outgoing neutron momentum) or of special assumptions, or of orthogonality.

Thus the matrix element is obtained mainly as a product of coefficients introduced by these expansions, such as various Clebsch-Gordan coefficients. Factors which are independent of angle, corresponding to the factor P_{ℓ_c} in the cross-section formula (C1), are replaced by an undetermined proportionality constant.

The matrix element for heavy-particle stripping is evaluated in the same way, introducing another undetermined proportionality constant. According to G1, these are combined into a total matrix element T for insertion into B1 or D2, after which the averaging over initial and summing over final quantum numbers must be performed.

The ratio of the two undetermined proportionality constants mentioned above is not evaluated but is adjusted to give the best fit to the experimental data. This will be discussed below (Section J). The stripping radius for each of the two modes is also kept as an adjustable parameter, and to obtain the good fits to experimental data which have been claimed, values of R ranging from 1.0 to 7.2 fermis have been quoted in various JohnsHopkins publications. These three adjustable parameters do not influence only the detailed structure of the curves obtained; on the contrary, their variation within ranges of values as reasonable as the values actually quoted can modify the appearance of the curves very considerably. While it is traditional in stripping theory to leave one continuous variable, the radius, as an adjustable parameter, one does not normally accept quite that much leeway in values of R.

The other ad hoc assumptions, approximations, and adjustable parameters introduced will now be listed, to emphasize the very considerable arbitrariness present in two-mode stripping theory. It should be borne in mind that all this is in addition to the assumptions and approximations inherent in stripping theory itself. As discussed elsewhere in this thesis, the criteria for their applicability in this particular case, even for pure deuteron stripping, are hardly satisfied in a convincing manner; their application to heavy-particle stripping is untested and even more debatable.

The Bhatia derivation involved a sum over the important angular momentum quantum number ℓ_c, and it turned out (Section E) that only the values $\ell_c = 1$ and 3 are allowed by conservation laws, in the deuteron stripping treatment of this reaction. Because it is quite reasonable, in shell-model terminology, to think of the proton as going into the available space in the p 3/2 level of B^{11} to form the first excited C^{12*}, the smaller of these values of ℓ_c is certainly the more important, as it is in most reactions. Nevertheless the Born approximation formula includes terms corresponding to both values. In two-mode stripping theory on the other hand, such a summation also occurs in the evaluation of the deuteron stripping matrix element, but it is arbitrarily reduced to the single term $\ell_c = 1$, suppressing the $\ell_c = 3$ contribution entirely.

This approximation seems questionable. Not that $\ell_c = 3$ is of comparable importance to $\ell_c = 1$ of course — the shell-model

argument is sufficiently decisive on this point. But if one is going to take into account the heavy-particle stripping picture of the C^{12*} configuration consisting of a point deuteron with orbital angular momentum 0 or 2 around a B^{10} core, the contribution of a proton with $\ell_c = 3$ may not be negligible in comparison with the latter picture, particularly since the $\ell_c = 3$ contribution is more important at large θ and at large radius R (as can be seen by comparing graphs of $\Pi(K_b) j_3^2(kR)$ for various R, to be presented in Chapter 3); the deuteron stripping radius quoted in the Johns Hopkins papers (7) is 6.8 fermis.

The same sum over ℓ_c, arbitrarily reduced to one term, occurs also in the evaluation of the matrix element for heavy-particle stripping. Here the angular momentum and parity selection rules allow $\ell_c = 0$, 2, 4, or 6. This refers to the orbital angular momentum of the B^{10} with respect to the deuteron, and is designated as $\ell(C)$ in the Johns Hopkins papers. The fits to experiment claimed by Ames and Owen (17) and quoted by Edwards (7) were for $\ell(C) = 0$; however a note added in proof to (7) states that "Recent experiments of Rask, Ames, and Owen [...] require that $\ell(C) = 2$ rather than $\ell(C) = 1$ [the latter is a misprint for $\ell(C) = 0$)] as used in the foregoing [...]". It is obvious that this quantity is being used as yet another adjustable parameter, one that can also produce significant changes in the angular distributions and correlations.

Moreover these experimental fits require an assumed spin of 1 for the B^{10} core, whereas the measured spin of the B^{10} nucleus is 3. If such an assumption is justifiable at all, it at least implies that this quantity is also being used as an adjustable parameter.

The evaluation of the factor $\Pi(K_b)$, the momentum transform (C2 and B7) of the deuteron's internal wave function, is quite approximate, since it is based on the zero-range wave function only. A more accurate form is the Hulthén wave function

$$\psi(r) \propto \frac{1}{r}\left[e^{-\alpha r} - e^{-\beta r}\right]$$

with $\beta \simeq 5.2\,\alpha$ (Moravcsik, 20). $\Pi(K_b)$ is then proportional to $\left[(K_b^2+\alpha^2)(K_b^2+\beta^2)\right]^{-2}$ instead of just $(K_b^2+\alpha^2)^{-2}$. The difference is unimportant when one is concerned only with the position of the principal maximum, but becomes significant for a study of the whole range of θ from 0 to 180°, especially at 2 Mev. A graph of this quantity $\Pi(K_b)$ in arbitrary units, plotted as a function of k, is given in Figure 5.

Much more uncertain is the corresponding factor $\Pi(K_b)$ which appears in the heavy-particle stripping matrix element. This is the momentum transform of the wave function for the p-shell outer neutrons in B^{11}. Owen and Madansky (12) have calculated this assuming a square well potential, and this is the result used in all Johns Hopkins publications. On this point it is possible that independent experimental evidence will become available, from measurements of (p,2p) quasi-elastic

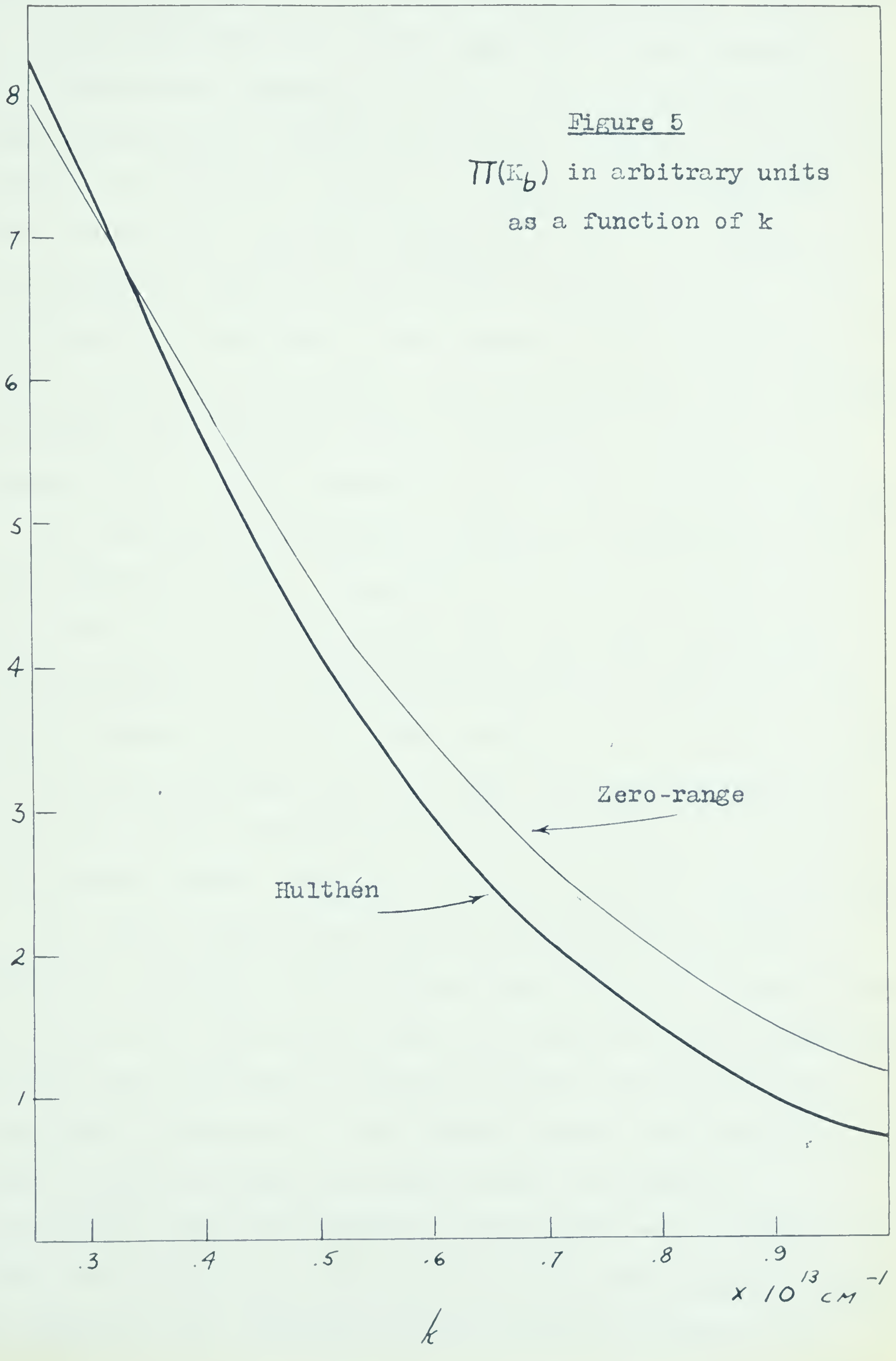


Figure 5
$\Pi(K_b)$ in arbitrary units
as a function of k
Zero-range
Hulthén
8
7
6
5
4
3
2
1
.3
.4
.5
.6
.7
.8
.9
x 10^{13} cm^{-1}
k

scattering performed with high-energy accelerators. (Wattenberg 21, Wilcox and Moyer 22). These experiments yield momentum distributions for protons in nuclei; some of the results have been interpreted as showing two groups of protons, namely core protons and peripheral protons. Of course the analysis of these experiments involves assumptions (e.g. the impulse approximation), and the deduction of neutron momentum distributions from these proton momentum distributions will require careful justification; but it is possible that relevant information may come out of this experimental method in the future, after a larger number of nuclei are studied in this way.

To continue the listing of adjustable parameters in two-mode stripping theory, it must be pointed out that in the evaluation of the matrix element for each mode one has to choose between the various coupling schemes possible for the angular momenta involved in the reaction. What happens is that if the wrong coupling scheme is used, expansions are made in terms of a set of eigenfunctions of a quantity which is not a good quantum number, and hence the expansions are inconvenient.

The final expressions obtained are sensitive to the particular coupling scheme chosen. These coupling schemes, even if two or three alternative ones are considered, are themselves extreme idealizations of the real physical situation. This is recognized by Edwards (7) who claims without proof that in the general case of a mixed coupling scheme, the angular distributions and neutron-gamma correlations must be fitted by linear combinations of the expressions for each coupling scheme, and suggests that a measure of the amount of mixing of the

coupling schemes can be obtained by treating the coefficients in these linear combinations as adjustable parameters to fit the experimental results. In view of the large number of other adjustable parameters present in the analysis, this suggestion must not be taken too seriously.

The measured angular correlations (Neilson et al., 9) can not be fitted by this theory, at least for any of the pure coupling schemes.

J- Relative Magnitudes

The most important weakness of two-mode stripping theory is that the parameter expressing the relative importance of the two modes is left open for adjustment to experimental data. This is objectionable because variation of this parameter within a range of values which, in the absence of any published theoretical estimate whatever, might be considered acceptable, can very significantly change the appearance of the curves. But the main objection to this procedure is the following. As discussed in Section F of this chapter, no one can object to heavy-particle stripping in principle; the only question, the crucial question, is whether there is reason to believe that in this particular reaction and a few others it should turn out to be as important as deuteron stripping, whereas its relative importance is normally negligible. By leaving this relative importance as an open parameter, no progress is made towards

answering this question, since the situation is otherwise too indeterminate — involves too many other adjustable parameters and assumptions — to say that the relative importance parameter is "determined"by this best-fit adjustment.

Information which might lead to estimates of the relative importance of the two modes is very sparse, mainly because one does not normally think in terms of point deuterons entering the configurations of nuclear states.

Hadley and York (23) have bombarded a C^{12} target with 90 Mev neutrons, and obtained both d + B^{11} and t + B^{10} as reaction products, the yield of tritons being at most 10% of the yield of deuterons. This can be explained as a pick-up process (Chew and Goldberger, 24), and hence a careful analysis might show to what extent the C^{12} ground state can be thought of as consisting of a deuteron plus B^{10} core rather than a proton plus B^{11}. Unfortunately we are concerned with an excited state of C^{12} rather than the ground state, and no such source of information is possible for excited states.

The description of the first excited state of C^{12} in terms of single proton excitation from the p 3/2 to the p $\frac{1}{2}$ level is acceptable but far from unique. The collective model also accounts for ground states of 0^+ and first excited states of 2^+ for even-even nuclei. Even an alpha-particle model for C^{12} satisfactorily correlates the ground state, the 2^+ first excited state, the 0^+ level at 7.65 Mev and the level at 9.61 Mev (Glassgold and Galonsky 25, as discussed in Almqvist et al., 26).

Holmgren et al. (27) have studied the reaction $B^{11}(He^3,d)C^{12*}$ (first excited state), and report some disagreement with the predictions of forward stripping theory. They state that the nature of the additional process involved can not be predicted on the basis of their present results, and after private communication with the Johns Hopkins group they conjecture that it may arise from exchange stripping or compound nucleus formation. They contend that "The enhancement of this process [heavy-particle stripping] relative to the forward stripping process in reactions leading to the first excited state may possibly be due to the 'cluster' characteristics (Wildermuth and Kanellopoulos [28]) of the low-lying excited states of C^{12}."

However the work they refer to (28) does not support this at all: it can be inferred from suggestions made in (28) that these authors (Wildermuth and Kanellopoulos) expect any states involving clusters smaller than alpha particles to be of negligible importance in C^{12}. Indeed the guiding principle of the cluster model seems to be that only those clusters turn out to be important which correspond to existing nuclei of exceptionally tight binding, such as alpha particles. This of course argues against the presence of a deuteron cluster in C^{12*}, and the success of the cluster model, really an alpha-particle model in this case, in dealing with C^{12*} is an argument against the enhancement of heavy-particle stripping.

In the absence of anything more definite, the following classical argument, suggested by Dr Trainor (29), provides some estimate of relative importance. Consider the break-up of

either the deuteron or the B^{11} to be produced by the impact of their collision. The work done by this impact is given by the product of the change in momentum by the change in velocity. Looking at the problem in the center-of-mass system, the two colliding systems suffer the same change in momentum; but the change in velocity is much more severe for the deuteron than for the B^{11}, (by a factor of 11/2), because of its smaller mass. Thus $5\frac{1}{2}$ times as much of the work done by the impact of the collision is available to the deuteron, for breaking bonds, as to the B^{11}.

In the case of the deuteron this energy of impact acts entirely on the single bond, but in the case of B^{11} it is divided among at least the four bonds holding the neutrons we are interested in; hence twenty-two times as much energy is available, on the average, for breaking the deuteron bond as for breaking any one of the four p-shell neutrons' bond in B^{11}.

These quantities of energy must be compared with the binding energy in each case. This introduces another factor of 5.2 in favor of the deuteron, since the binding energy of the last neutron in B^{11} is 11.5 Mev as compared to 2.226 Mev for the deuteron. Thus at 2 Mev bombarding energy for example, $\Delta p.\Delta v = \hbar^2 k_d^2/m$ is about 2.88 Mev for the deuteron, compared to a binding energy of 2.226 Mev; whereas it is only about .524 Mev for B^{11}, or at most .13 Mev per bond on the average, compared to a binding energy of 11.5 Mev. Hence a Break-up of B^{11} leading to heavy-particle stripping is much less probable than a break-up of the deuteron leading to deuteron stripping.

In addition to this enormous advantage for deuteron stripping relative to heavy-particle stripping, there must also be an additional advantage coming from the reduced widths of the final state, since $B^{11} + p$ is certainly a better picture of C^{12*} than $B^{10} + d$, even if it is difficult to specify how much better. Admittedly these calculations are based on a very inadequate model of the stripping process, but at least they illustrate why one is reluctant to consider heavy-particle stripping to be as important as deuteron stripping.

The general conclusion that seems to emerge from all this is that no valid reason whatever has been put forward to expect the relative importance of heavy-particle stripping relative to deuteron stripping to be greater in this reaction and a few others than it normally is; and that the analysis performed on this assumption involves too many approximations and adjustable parameters for the fits to experiment to be considered as verifying it.

This does not detract from the value of the work reviewed in showing how the complicated kinematics may be handled and the two modes formally combined. The analysis may be of value in another reaction, or under experimental conditions such that one can be sure that no trouble arises from Coulomb or nuclear effects neglected by the basic stripping approximations.

Unfortunately the Be^9 nucleus, which, because of the very low binding energy of its last neutron (1.7 Mev), would seem to be the most favorable subject for a heavy-particle stripping study, is likely to be very complicated to study because it

seems that one should really consider it as two alpha-particle clusters plus an extra neutron, rather than a Be^8 core plus a neutron. Moreover, the work of Neilson et al. (9) on this nucleus shows angular distributions which, for all but one of the states of B^{10}, do not resemble the curves predicted by heavy-particle stripping theory.

In high-energy bombardment of deuterons by deuterons, of course, two modes must be considered.

Finally, one must remember in this type of work that the interference term may be quite large even though the pure heavy-particle stripping term is negligible.

Chapter 3 ALTERNATIVE EXPLANATIONS

K- Qualitative Discussion

The special features of the neutron angular distributions observed experimentally for B^{11} (d,n)C^{12*}(4.43 Mev state) and other similar reactions have been described in Section E, and some shortcomings of the heavy-particle stripping proposal as a detailed explanation of them have been indicated in Chapter 2. In the present work, a study has been made of the possibility of accounting for these special features, in particular the large cross-sections for $\theta > 90^\circ$, without introducing the heavy-particle stripping hypothesis.

It is not contended that heavy-particle stripping does not take place, for in principle of course such effects must be considered. However the work of the Johns Hopkins group (Chapter 2) which attempts to explain the backward distributions as a heavy-particle stripping effect suggests that heavy-particle stripping contributes equally with deuteron stripping. In the present chapter, the point of view is adopted that heavy-particle stripping contributes only in a minor way, and that one must seek elsewhere for an explanation of the backward distributions.

One suggested alternative explanation can be summarily dealt with. Because some results for Si^{28}(d,p)Si^{29} and Al^{27}(d,p)Al^{28}, also showing abnormally high cross-section at backward angles, seemed well fitted by stripping curves with

the factor $\Pi(K_b)$ replaced by a constant, Bromley et al. (44) have examined a number of deuteron potentials, and demonstrated that no reasonable potential will produce such a constant form factor.

There is reason to think that Coulomb effects, not considered at all in the basic theory of deuteron stripping, may be particularly important in this reaction. Criteria designed to establish whether their neglect is justified in particular cases (Butler, 1) are not satisfied for this reaction at the bombarding energies used, since the Coulomb barrier as seen by the approaching deuteron is of the same order of magnitude as these bombarding energies. The reason one tries to fit results on the basis of stripping theory at all is that in other cases it often seems to work reasonably well even when the criteria are not satisfied. The procedure of keeping the stripping radius as an adjustable parameter to obtain the best fit to experiment seems to cover a multitude of sins. But as the particular results under study turn out to be very different from the usual stripping distributions, it is natural to ask whether one can see in a simple way if Coulomb effects might modify the distributions in the required way.

Qualitatively, this is confirmed by descriptive arguments, which vary depending on how one chooses to look at the process. For a (d,n) reaction the standard qualitative description (e.g. Butler (1), Chapter 1) considers separately the effect of Coulomb forces on the trajectory of the approaching deuteron, and on the wave function describing the proton after capture by

the nucleus. The latter effect can be seen to lead to a considerable reduction in the total cross-section but has little effect if any on the angular distribution. It can be thought of as acting only on the capture probability P_{ℓ_c} of C1, which is independent of angle, and is therefore of no interest for present purposes. The Coulomb effect on the deuteron trajectory, on the other hand, is interesting, and even a purely classical description of it brings out the essential features of the modifications thus introduced.

Classically a deuteron approaching with kinetic energy E a center of repulsive Coulomb force Ze^2/r^2 at an impact parameter b, suffers an angular deflection of δ away from the incident direction, where

$$\tan\frac{\delta}{2} = \frac{Ze^2}{bE}.$$

If a stripping process occurs which would otherwise have produced an outgoing neutron at an angle θ, the result roughly is an outgoing neutron at an angle $\theta' = \theta + \delta$, since the Coulomb field has already led to a deflection δ.

A good idea of the resulting neutron angular distribution can therefore be obtained by just replacing θ by θ' in the uncorrected angular distribution, with b of the order of the stripping radius R or somewhat greater. Thus peaks and minima of the angular distribution are shifted to larger scattering angles by a constant amount when Coulomb forces are taken into account in this crude manner, the amount decreasing as the bombarding energy increases relative to the Coulomb barrier

height. Moreover, since many different impact parameters simultaneously contribute, it is evident that the sharp features of the stripping pattern are smeared out or attenuated to some extent.

Now the observed anomalous angular distributions for $B^{11}(d,n)C^{12*}$ might be described as an extreme case of such a shift in pattern and attenuation of features. Of course the classical argument above can only indicate the necessity of a proper calculation of the Coulomb effect. This turns out to be extremely difficult to do.

L- Coulomb Functions

This section introduces the mathematical functions which have been found necessary both in the standard treatments of Coulomb effects in stripping (Section M) and in the different treatment proposed here (Section N).

The approach of a particle of reduced mass m and charge +e, with momentum $k = mv$ with respect to a scattering center of charge Ze, is governed by the Schrödinger equation

$$\nabla^2 \psi + \left(k^2 - \frac{2m}{\hbar^2}\frac{Ze^2}{r}\right)\psi = 0 \qquad \text{L1}$$

A solution of this equation which describes the scattering in a physically acceptable way can be found (Mott and Massey, 30) by making the substitution $\psi = e^{ikz} F$, making use of the fact that $F = F(r-z)$. The Frobenius method of solving the

differential equation then leads to

$$\psi_G(r,\theta) = e^{-\pi\eta/2}\,\Gamma(1+i\eta)\, e^{ikr\cos\theta}\, {}_1F_1(-i\eta; 1; ikr(1-\cos\theta)) \qquad \underline{\text{L2}}$$

where

$$\eta = \frac{Ze^2}{\hbar v} = \frac{Ze^2 m}{\hbar^2 k},$$

and $\Gamma(x)$ is the gamma function. The function

$${}_1F_1(a; b; z) = 1 + \frac{a}{b} z + \frac{a(a+1)}{b(b+1)} \frac{z^2}{2!} + \cdots$$

is called the confluent hypergeometric function (or series) by many authors (Jahnke and Emde, 31; Bloch et al., 32), and the Kummer function by others (Magnus and Oberhettinger, 33).

It is more useful for many purposes to solve L1 by separation of variables in spherical polar coordinates. This procedure leads to a radial equation

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dL}{dr}\right) + \left(k^2 - \frac{2k\eta}{r} - \frac{\ell(\ell+1)}{r^2}\right) L = 0 \qquad \underline{\text{L3}}$$

A solution of this can be written in terms of confluent hypergeometric functions different from those used above:

$$L_\ell(r) = e^{-\pi\eta/2} \frac{|\Gamma(\ell+1+i\eta)|}{(2\ell+1)!} (2kr)^\ell e^{ikr}\, {}_1F_1(i\eta+\ell+1; 2\ell+2; -2ikr)$$

These solutions $L_\ell(r)$ occur as coefficients in the very important spherical harmonics expansion of $\psi_G(r, \theta)$ established by Gordon (34):

$$\psi_G(r,\theta) = \sum_{\ell=0}^{\infty} (2\ell+1)\, i^\ell e^{i\sigma_\ell} L_\ell(r)\, P_\ell(\cos\theta) \qquad \underline{\text{L4}}$$

where $\sigma_\ell = \arg\Gamma(\ell+1+i\eta)$.

One prefers, nowadays, to transform the radial equation L3 by introducing the new variables $\rho = kr$ and $u(\rho) = \rho L(\rho)$. This gives

$$\frac{d^2 u}{d\rho^2} + \left[1 - \frac{2\eta}{\rho} - \frac{\ell(\ell+1)}{\rho^2}\right] u = 0$$

Two independent solutions of this last form of the equation are known as the Coulomb functions. They are: the regular function $F_\ell(\rho)$, a real function which vanishes at $\rho = 0$ and is normalized so that its asymptotic behaviour for $\rho \gg \ell$ is

$$F_\ell(\rho) \cong \sin\left[\rho - \frac{\ell\pi}{2} - \eta \ln(2\rho) + \sigma_\ell\right] ;$$

and the irregular solution $G_\ell(\rho)$ which does not vanish at $\rho = 0$, and has the corresponding asymptotic form

$$G_\ell(\rho) \cong \cos\left[\rho - \frac{\ell\pi}{2} - \eta \ln(2\rho) + \sigma_\ell\right] .$$

This normalization implies that the Wronskian

$$G_\ell \frac{dF_\ell}{d\rho} - F_\ell \frac{dG_\ell}{d\rho} = 1 .$$

$F_\ell(\rho)$ and $G_\ell(\rho)$ are of course functions of the variable η which measures the strength of the Coulomb interaction.

M- Coulomb Effects in Stripping

The introduction of the effects of Coulomb forces into the standard theory of stripping angular distributions has been studied by many authors (e.g. 34 to 39). The following features are typical of such treatments.

Calculations are performed for the (d,p) reaction, a brief footnote usually claiming without discussion that they can be applied, mutatis mutandis, to the (d,n) reaction also.

The method is simple in principle: the plane waves describing the approaching deuteron (considered as a point particle) and the outgoing proton are replaced by the appropriate Coulomb-distorted wave functions $\psi_G(r, \theta)$ (see previous section), and the analysis is carried through along the general lines of the non-Coulomb treatment, except that the simplicity and significance of the derivation as presented in Chapter 1 are lost in the process.

These functions $\psi_G(r, \theta)$ have to be expressed for instance as contour integrals in the complex plane, whose arguments are expanded in various ways, leading to a formidable amount of numerical analysis and computation, much of which must be performed anew for each particular reaction studied.

These calculations have been performed for several particular cases, and the results obtained generally do confirm the three trends brought out qualitatively in Section K, i.e. reduction in absolute cross-section, shift of the recognizable stripping pattern towards larger θ, attenuation of the sharpness of its features; and they generally fit experimental results somewhat better than the uncorrected formulas.

These calculations treat the deuteron as a point charge, rather than recognizing the fact that the Coulomb field really acts on the proton. Some treatments attempt to alleviate this in an uncertain way by using a wave function which represents a

particle having the deuteron's mass and charge but the position coordinates of the proton.

A full treatment of the stripping problem would also include corrections for the complex nuclear interactions involved, which are largely neglected by the basic stripping assumptions. This can not be done adequately since nothing but a few generalities are known unequivocally about these interactions, as well as about detailed nuclear structure. A few calculations have been made which try to take the nuclear interactions into account by introducing simple models, for particular cases, in combination with Coulomb effect calculations mentioned above; the same comments apply. The problem reduces in principle to a determination of the amount of irregular Coulomb function $G_\ell(\rho)$ to be properly mixed in with the regular $F_\ell(\rho)$; in practice this tends to complicate the already severe computational problem by orders of magnitude considering the various possible models to be considered and the additional unknown parameters involved. The trend of results, for those cases which have been considered, is to cancel out part of the pure Coulomb modifications outlined above. For practical purposes it seems that a good case can be made for considering Coulomb corrections while still neglecting nuclear corrections, unless one is very close to a compound nucleus resonance level. (Resonances do occur in reactions which show good stripping angular distributions.)

N- Proposal for (d,n) Reactions

From the point of view of trying to explain the $B^{11}(d,n)C^{12*}$ anomalous angular distributions as a special Coulomb effect, the work mentioned in the last section is not very helpful. The computations are too complicated. It was designed to deal with cases where the characteristic stripping pattern is somewhat modified, not essentially transformed. And it was designed to deal with (d,p) reactions rather than (d,n).

The last point is particularly significant. It may be adequate to consider Coulomb effects on a point deuteron when one is discussing (d,p) stripping, in which the proton never comes close enough to the target nucleus to feel the full force of the Coulomb interaction, although there is some current uneasiness about this.† However, there is much less justification for this approach in (d,n) reactions, particularly if one is looking specifically for significant modifications of the stripping angular distributions due to Coulomb effects on the close approach of the charged proton.

This should be made more forceful by another version of the qualitative argument which leads us to expect that a strong

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† As evidenced for instance by this statement from the latest major work on the subject (Satchler and Tobocman, 39): " it is not certain [...] even that merely distorting the center-of-mass motion of the deuterons is an adequate representation of their wave function close to the nucleus."

Coulomb effect can favor large angle stripping in a (d,n) reaction. Consider the problem in terms of barrier penetration. Barrier penetration by the deuteron is not a very helpful concept in this context because there is no unique radius to which the deuteron must penetrate for stripping to take place. But barrier penetration by the proton is more informative, since the proton must go in far enough to be stripped off by nuclear forces. Now the probability of penetration of a Coulomb barrier is a very sensitive increasing function of the momentum of approach. The presence of the barrier thus favors high momentum of approach k and hence large angle θ. It is obvious that to study such an effect quantitatively one must consider Coulomb effects on the proton's approach to the nucleus with momentum k rather than on the whole deuteron.

We are dealing here with a quantum mechanical three-body problem involving proton, neutron, and target, which must be reduced to a more amenable problem by one trick or another. The work appearing in the literature to date has reduced it first to a two-body problem involving point-deuteron and target. Such approaches do not do justice to the physical process described in the previous paragraph. An alternative approach to the reduction of the three-body problem is considered here. The point is that in the case under study, the deuteron bombarding energy, the binding energy of the deuteron, and the Coulomb and centrifugal potentials of consequence between proton and target are all of the same order of magnitude, about $\frac{1}{2}$ to 5 Mev, so that alternative reductions may be equally justifiable.

Consider the three-factor formula C1

$$\sigma = \Pi(K_b) \sum_{\ell_c} L_{\ell_c}(k)\, P_{\ell_c}$$

discussed extensively in Section C. Ignoring entirely Coulomb effects on the deuteron as such, let us try simply to replace the factor $L_{\ell_c}(k)$, describing the approach of the proton towards the target as a plane wave with momentum k, by a corresponding factor $L'_{\ell_c}(k)$ describing as nearly as possible the same situation with Coulomb forces included.

The factor $L_{\ell_c}(k)$ is the significant one, which determines the characteristic features of the usual stripping. It seems reasonable that modification of the factor $L_{\ell_c}(k)$ should account for the major Coulomb effects on the close approach of the proton leading to its absorption into the target, whereas the arguments advanced to justify the standard Coulomb treatment of (d,p) reactions (Section M) still have some validity for description of the distant approach of the whole deuteron. These arguments hold that because the Coulomb field does not change too drastically during the time corresponding to the internal period of the deuteron, the deuteron's internal motion, and hence $\Pi(K_b)$, is not too affected by them. Coulomb effects on the factor P_{ℓ_c} do not affect the angular distribution, and can thus be left out.

The appropriate form of $L'_{\ell_c}(k)$ has been deduced by analogy with the simple Born approximation formula developed and discussed extensively in Chapter 1. There $L_{\ell_c}(k)$ arises as the modulus squared of the coefficient in the spherical harmonics expansion B8 of a plane wave representing the approach of

proton to target with relative momentum k. Retaining as much as possible of this picture of the break-up of the deuteron producing such a plane wave, we consider what is the nearest thing to a plane wave possible in the presence of Coulomb forces. This is the function (L2)

$$\psi_G(r,\theta) = e^{-\pi\eta/2}\,\Gamma(1+i\eta)\, e^{ikr\cos\theta}\; {}_1F_1(-i\eta;1;ikr(1-\cos\theta)).$$

For the particular case of vanishing Coulomb forces, $\eta = 0$, the differential equation L1 satisfied by $\psi_G(r,\theta)$ reduces to the free-particle Schrödinger equation

$$\nabla^2\psi + k^2\psi = 0$$

of which the plane wave $e^{ikr\cos\theta}$ is a solution, and each of the factors $e^{-\pi\eta/2}$, $\Gamma(1+i\eta)$ and ${}_1F_1$ reduces to unity, leaving only the plane wave $e^{ikr\cos\theta}$.

Moreover the Gordon expansion L4 of $\psi_G(r,\theta)$, in modern notation

$$\psi_G(r,\theta) = \sum_{\ell=0}^{\infty} i^{\ell}\sqrt{4\pi(2\ell+1)}\; e^{i\sigma_\ell}\,\frac{F_\ell(\rho)}{\rho}\; Y_\ell^0(\theta),$$

corresponds precisely to the expansion B8 of a plane wave in spherical harmonics. The $F_\ell(\rho)/\rho$ are solutions of the radial Coulomb equation

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dL}{dr}\right) + \left(k^2 - \frac{2k\eta}{r} - \frac{\ell(\ell+1)}{r^2}\right)L = 0 \qquad \text{N1}$$

which reduces, for $\eta = 0$, to the equation

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dL}{dr}\right) + \left(k^2 - \frac{\ell(\ell+1)}{r^2}\right)L = 0 \qquad \text{N2}$$

of which the spherical Bessel functions $j_\ell(\rho)$ appearing in the uncorrected stripping formula are solutions. It follows that the required factor is

$$L'_{\ell_c}(k) = 4\pi(2\ell_c+1)\left(\frac{F_{\ell_c}(\rho)}{\rho}\right)^2$$

In addition to $F_{\ell_c}(\rho)/\rho$, equation N1 also has other solutions $G_{\ell_c}(\rho)/\rho$, irregular at the origin, just as equation N2 has solutions $n_\ell(\rho)$ in addition to $j_\ell(\rho)$. These are excluded from the study of many idealized physical problems because they diverge at the origin and are thus not physically acceptable. However in problems where the range of applicability of this differential equation does not include the origin, this objection does not arise and the general solution includes some contribution from the irregular solution as well, the relative amount depending on the boundary conditions. The problem of a particle approaching a nucleus is in fact such a problem, since the Schrödinger equation for the interior of the nucleus, which includes the origin of coordinates, is different from the simple Coulomb equation, and a fully rigorous solution (along the lines of the Butler approach if it did not involve subsequent approximations) would certainly involve the irregular solutions. But the Born approximation treatment, neglecting the effect on the plane wave of the presence of the nucleus, gives good results in the uncorrected stripping case without involving the irregular

spherical Neumann functions $n_\ell(\rho)$; it therefore seems reasonable to use this $L'_{\ell_c}(k)$ involving only the regular $F_{\ell_c}(\rho)$.

The proposed formula, then, is

$$\sigma(\theta) = \Pi(K_b) \sum_{\ell_c} L'_{\ell_c}(k)\, P_{\ell_c}, \qquad \text{N3}$$

with

$$L'_{\ell_c}(k) = 4\pi\,(2\ell_c+1)\left(\frac{F_{\ell_c}(\rho)}{\rho}\right)^2 \qquad \text{N4}$$

replacing C3

$$L_{\ell_c}(k) = 4\pi\,(2\ell_c+1)\, j_{\ell_c}^2(\rho).$$

P- A Method of Examining Reaction Data

In the course of this work a method of examining experimental angular distributions has been used which can be helpful and timesaving in deciding whether certain data exclude a pure deuteron stripping explanation or some simple modification of it such as the one suggested above. The method does not seem to have been pointed out explicitly before, although the approximations on which it is based are generally known and used in numerical calculations.

Again we use the three-factor cross-section formula C1,

$$\sigma = \Pi(K_b) \sum_{\ell_c} L_{\ell_c}(k)\, P_{\ell_c}$$

and neglect P_{ℓ_c} which is independent of angle. The most important factor, $L_{\ell_c}(k)$, depends on angle and on bombarding energy only insofar as it depends on k, which is given by

$$k^2 = k_d^2 + \left(\frac{m_A}{m_B} k_b\right)^2 - 2\frac{m_A}{m_B} k_b k_d \cos\theta . \qquad \text{P1}$$

As to the factor $\Pi(K_b)$, its dependence on θ and on energy is, strictly speaking, through the quantity K_b which is determined by

$$K_b^2 = k_b^2 + \tfrac{1}{4} k_d^2 - k_b k_d \cos\theta$$

But K_b is related by conservation laws to the momentum k, and it turns out that this relation is given to a good approximation by a formula†

$$K_b^2 + \alpha^2 \approx \frac{m_A + 1}{2m_A}(k^2 + \kappa^2) .$$

which does not otherwise involve bombarding energy or angle. I.e., for a given reaction to a given level of the final nucleus, all the quantities appearing in this approximate formula except K_b and k themselves are independent of angle and of bombarding energy. (α is the internal wave number related to the binding energy of the deuteron $\frac{\hbar^2\alpha^2}{2m^*}$, with $m^* = \frac{m_c m_b}{m_c + m_b}$; and κ is similarly related to the binding energy $\frac{\hbar^2\kappa^2}{2m_c^*}$ of the captured nucleon c in the final nucleus.) Thus the factor $\Pi(K_b)$ also, is expressed for purposes of computation as a function of k, which does not depend on angle except insofar as it depends on k. This is therefore true of the whole relative angular distribution expression.

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† Derived from an equation amounting to conservation of energy, by setting masses of nucleons and nuclei (in atomic mass units) equal to the nearest integer.

Suppose now that experimental data for the same reaction (same level) at different bombarding energies is available. If the stripping formula is to describe it, the variation of σ with θ at different energies should be such that the cross-section as a function of k be unique. The cross-section at one combination of energy-value and angle-value giving a certain k should be the same as that at any other combination of energy-value and angle-value giving the same k. Experimental curves of cross-section as a function of θ for different energies should therefore coalesce to a single curve when the points are replotted as a function of k instead of θ. Or at least their deviations therefrom should be random, not systematic.

Before applying this test the curve for each energy may be renormalized ad hoc since the cross-section measurements are only relative in any case. The variation of P_{ℓ_c} with bombarding energy is therefore irrelevant. Moreover an arbitrarily adjustable amount of isotropic background blamed on processes other than stripping may be subtracted from the experimental curves.

If the data are such that they can be fitted by the pure stripping formula C1 (either Born approximation or Butler), for any R, and for any combination of values of P_{ℓ_c}, (i.e. even for the unexpected case of $P_{\ell_c'} >> P_{\ell_c}$ where $\ell_c' > \ell_c$); or by the modified formula N3 embodying Coulomb effects as suggested in the preceding section; or by a similar formula designed to cope with other modifications within the framework of the very general formula

$$\sigma = \pi(K_b) \sum_{\ell_c} L''_{\ell_c}(k)\, P_{\ell_c}$$

as long as $L''_{\ell_c}(k)$ is a function of θ only through k; then a unique curve for $\sigma(k)$ will be obtained, and the task of looking for a theoretical expression to predict it should be facilitated by that uniqueness. The possibilities mentioned imply such a great variety of possible shapes, considering the great latitude currently fashionable in the choice of R, that merely inspecting curves of $\sigma(\theta)$ would not have helped to decide this.

If on the contrary the data include a significant non-isotropic contribution from processes other than deuteron stripping, such as for instance heavy-particle stripping, it is very unlikely that they should exhibit such uniqueness. One has then a valuable indication that such processes must be taken into account.

The procedure then, is first to calculate k_b and k_d for the various experimental bombarding energies, from the energetics of the reaction, then to tabulate or graph k as a function of θ (from Equation P1) for each energy.

It will be seen from P1 that the range $0 \leq \theta \leq 180°$ corresponds to greater and greater ranges of k as the bombarding energy increases; the range of k at a certain energy includes the ranges obtained for all lower energies. Using larger and larger bombarding energies allows us to look at a greater and greater portion of the curve of σ as a function of k.

One should then replot each curve $\sigma(\theta)$ as $\sigma(k)$ and renormalize so that the curves for different energies will all agree at a certain common value of k. If a unique curve is obtained in this way, it is most convenient at this point to try to reproduce it by starting from the theoretical end, plotting against k families of curves of $\pi(K_b)L_{\ell_c}(k)$ for various not too unreasonable values of R, and for the various allowed ℓ_c. It is surprising what a wide variety of curves can be predicted or "postdicted" in this way.

Q- Application to the Present Case

The experimental angular distributions of Neilson et al. (9) for the reaction $B^{11}(d,n)C^{12*}$(4.43 Mev level), at bombarding energies of $\frac{1}{2}$, 1, $1\frac{1}{2}$, and 2 Mev, have been analysed in this way, except that no attempt has been made to subtract some homogeneous background from each curve. (These results agree with those obtained by other groups, and which have been considered as one of the central pieces of evidence for the heavy-particle stripping hypothesis.) The curves obtained as a function of k after renormalization are shown in Figure 6.

It turns out that this is a borderline case with respect to the test described above. Remembering that the average ordinates have been renormalized ad hoc, so that only the agreement in shape and in abcissae of features tests the hypothesis, their coalescing to a single curve is not too

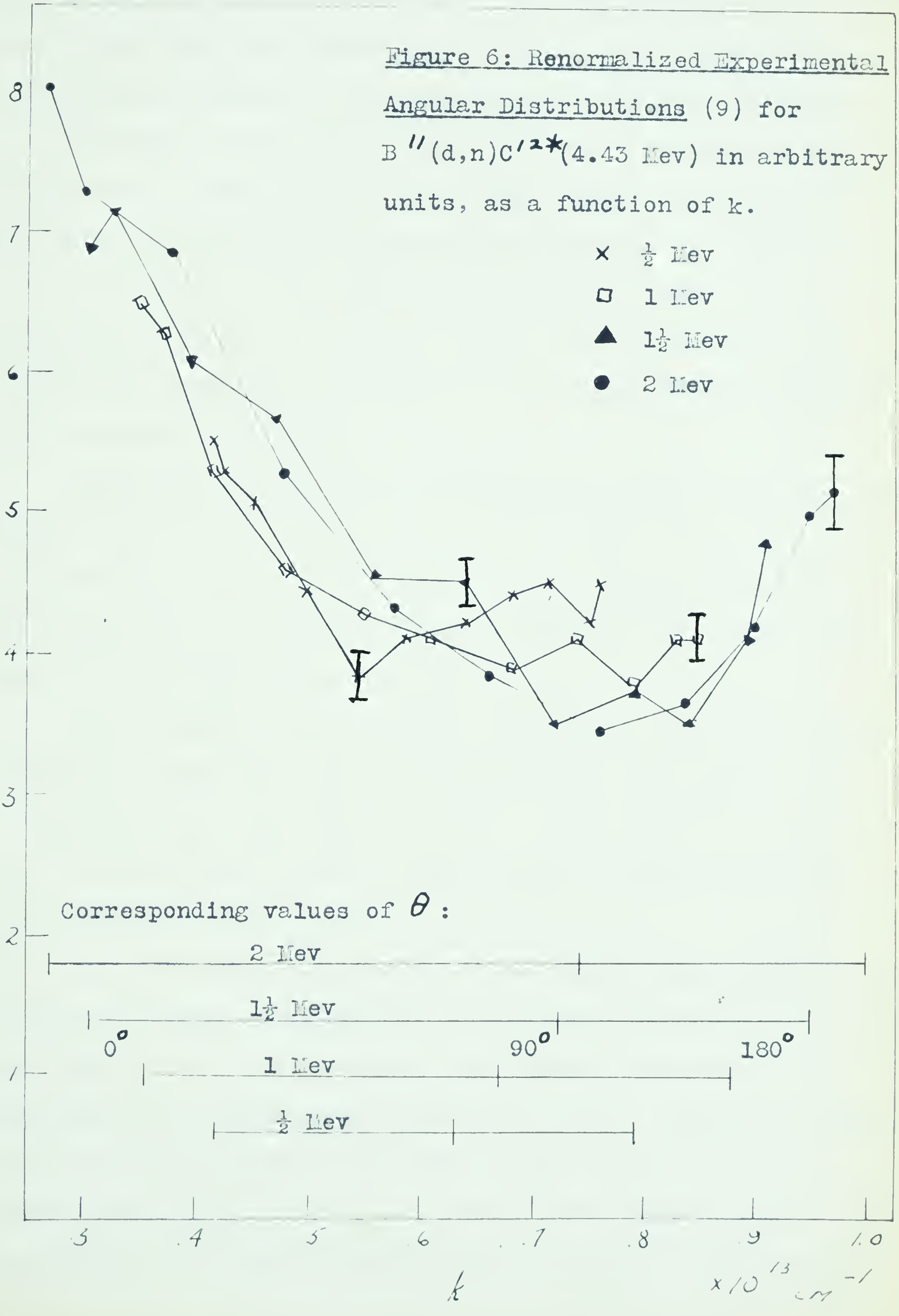
Figure 6: Renormalized Experimental Angular Distributions (9) for $B^{11}(d,n)C^{12*}$(4.43 Mev) in arbitrary units, as a function of k.
× ½ Mev
□ 1 Mev
▲ 1½ Mev
● 2 Mev
8
7
6
5
4
3
2
1
Corresponding values of θ :
2 Mev
1½ Mev
0°
90°
180°
1 Mev
½ Mev
.3
.4
.5
.6
.7
.8
.9
1.0
k
$\times 10^{13}$ cm^{-1}

convincing. But it is not so bad that if a suitable theoretical expression were put forward it could not conceivably be considered as agreeing approximately with all four of them.

Inasmuch as they can be described as coalescing to a single curve, this curve is described as follows: a smooth monotonic decline from the smallest k reached ($k = .27$), levelling off to a flat minimum at $k = .80$ to $.84$; then a gradual rise from $k = .84$ to $k = .97$. This is what must be predicted theoretically if one is to do without other processes than deuteron stripping.

Curves of the quantities $\pi(K_b)j_1^2(\rho)$ and $\pi(K_b)j_3^2(\rho)$ for radii R of 2, 3, 4, 5, 6, 7, 8, and 9 fermis have been plotted as functions of k, to decide whether an unexpected value of R or of the ratio P_3/P_1 might produce anything like the average experimental curve described above.

Following the proposal made in Section D for introducing Coulomb effects into (d,n) stripping theory, the quantities $\pi(K_b)F_1^2(\rho)/\rho^2$ and $\pi(K_b)F_3^2(\rho)/\rho^2$ have also been plotted for the same wide choice of radii. Some of these graphs are presented in Figures 7 to 10.

Detailed examination of all these curves has revealed that neither the pure stripping nor the Coulomb function curves, for any ratio P_3/P_1, can reproduce the general features of the aggregate experimental curve described above. Those few curves which show the required rise from $k \simeq .80$ to $.97$ all show a marked rise from $k = .27$ to a peak at some higher k, in contrast to the required smooth decline from $k = .27$ to $.80$.

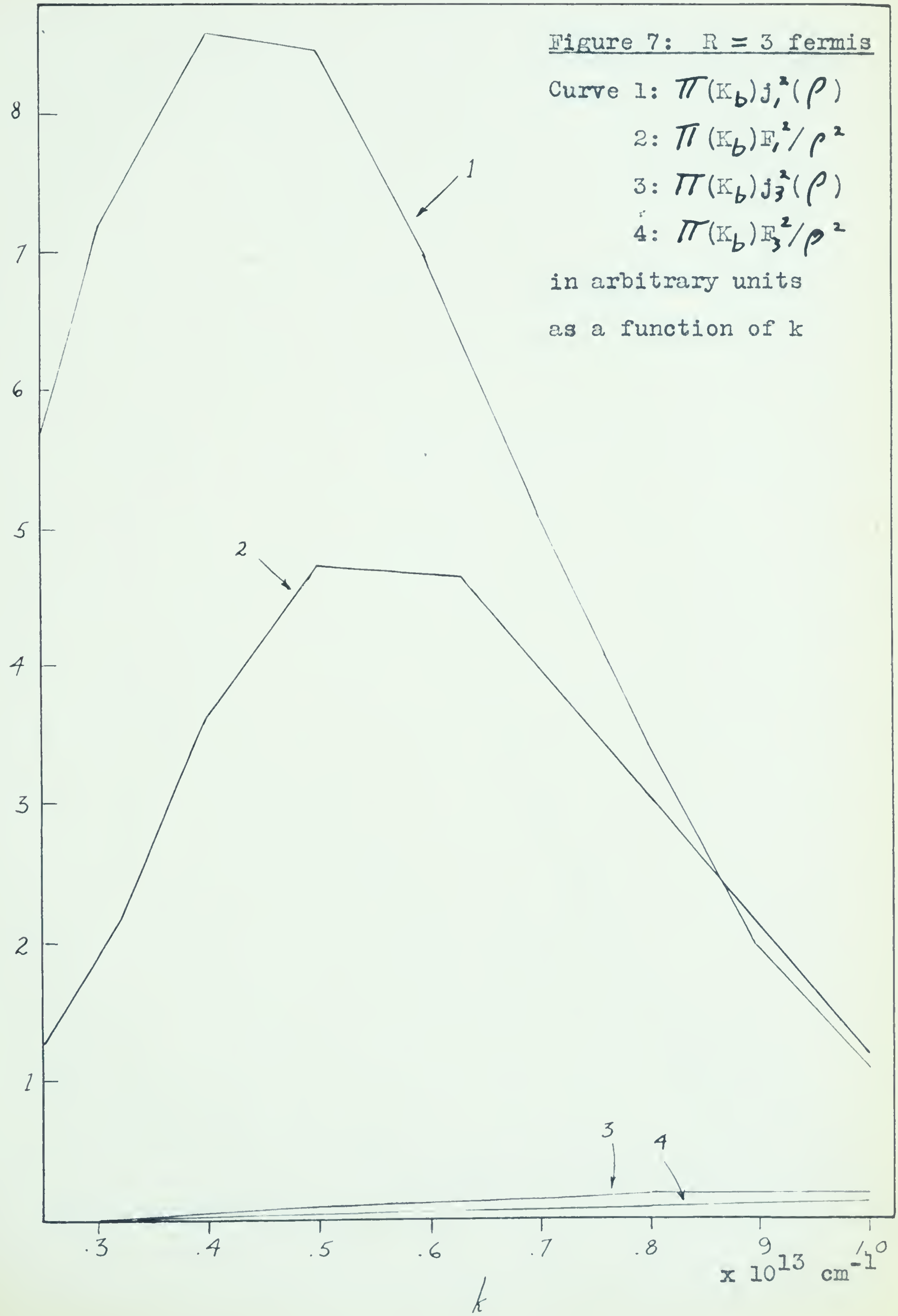
Figure 7: R = 3 fermis
Curve 1: $\pi(K_b) j_1^2(\rho)$
2: $\pi(K_b) E_1^2/\rho^2$
3: $\pi(K_b) j_3^2(\rho)$
4: $\pi(K_b) E_3^2/\rho^2$
in arbitrary units
as a function of k
1
2
3
4
8
7
6
5
4
3
2
1
.3
.4
.5
.6
.7
.8
.9
1.0
x 10^{13} cm^{-1}
k

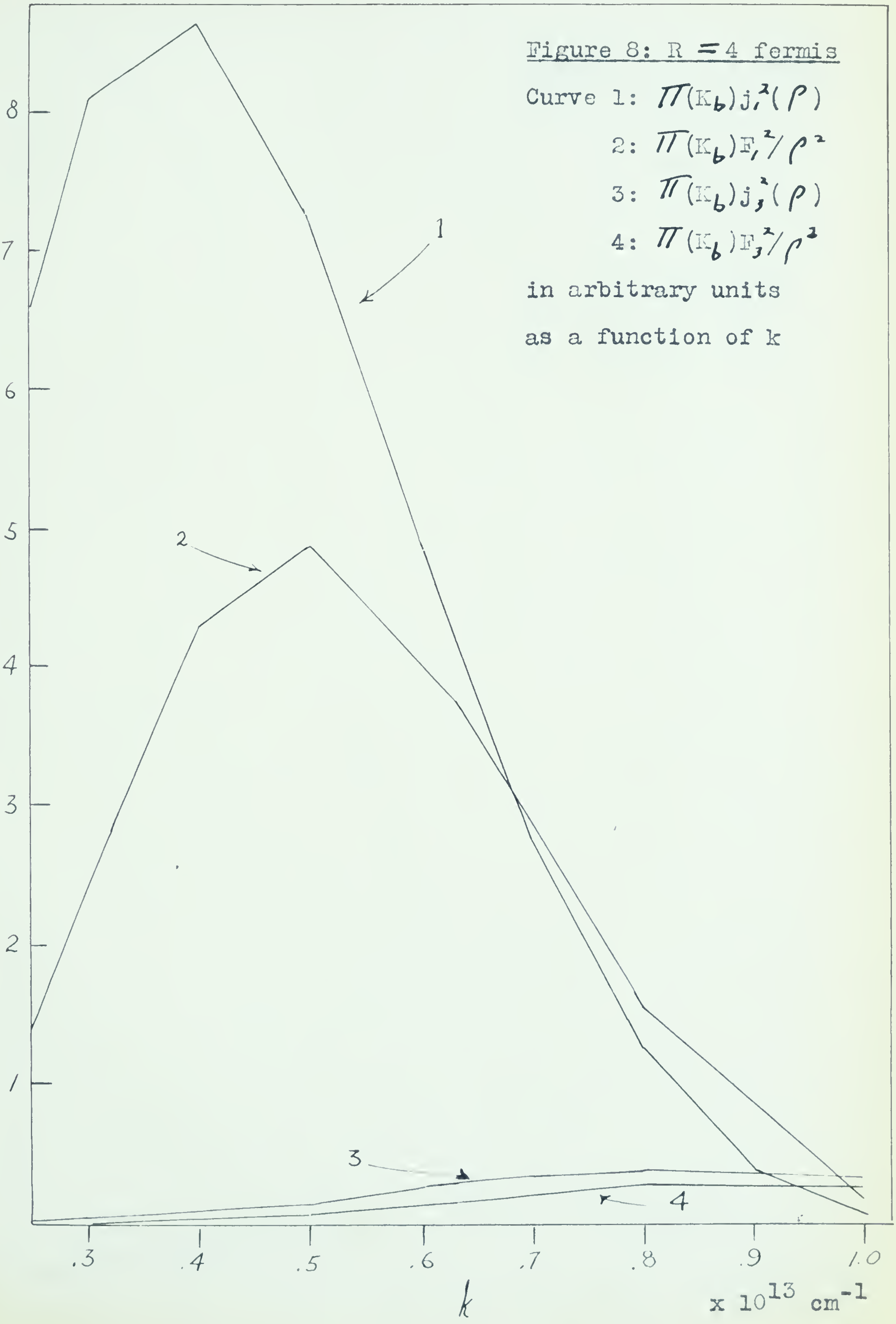
Figure 8: R = 4 fermis
Curve 1: $\pi(K_b)j_1^2(\rho)$
2: $\pi(K_b)F_1^2/\rho^2$
3: $\pi(K_b)j_3^2(\rho)$
4: $\pi(K_b)F_3^2/\rho^2$
in arbitrary units
as a function of k
1
2
3
4
8
7
6
5
4
3
2
1
.3
.4
.5
.6
.7
.8
.9
1.0
k
x 10^{13} cm^{-1}

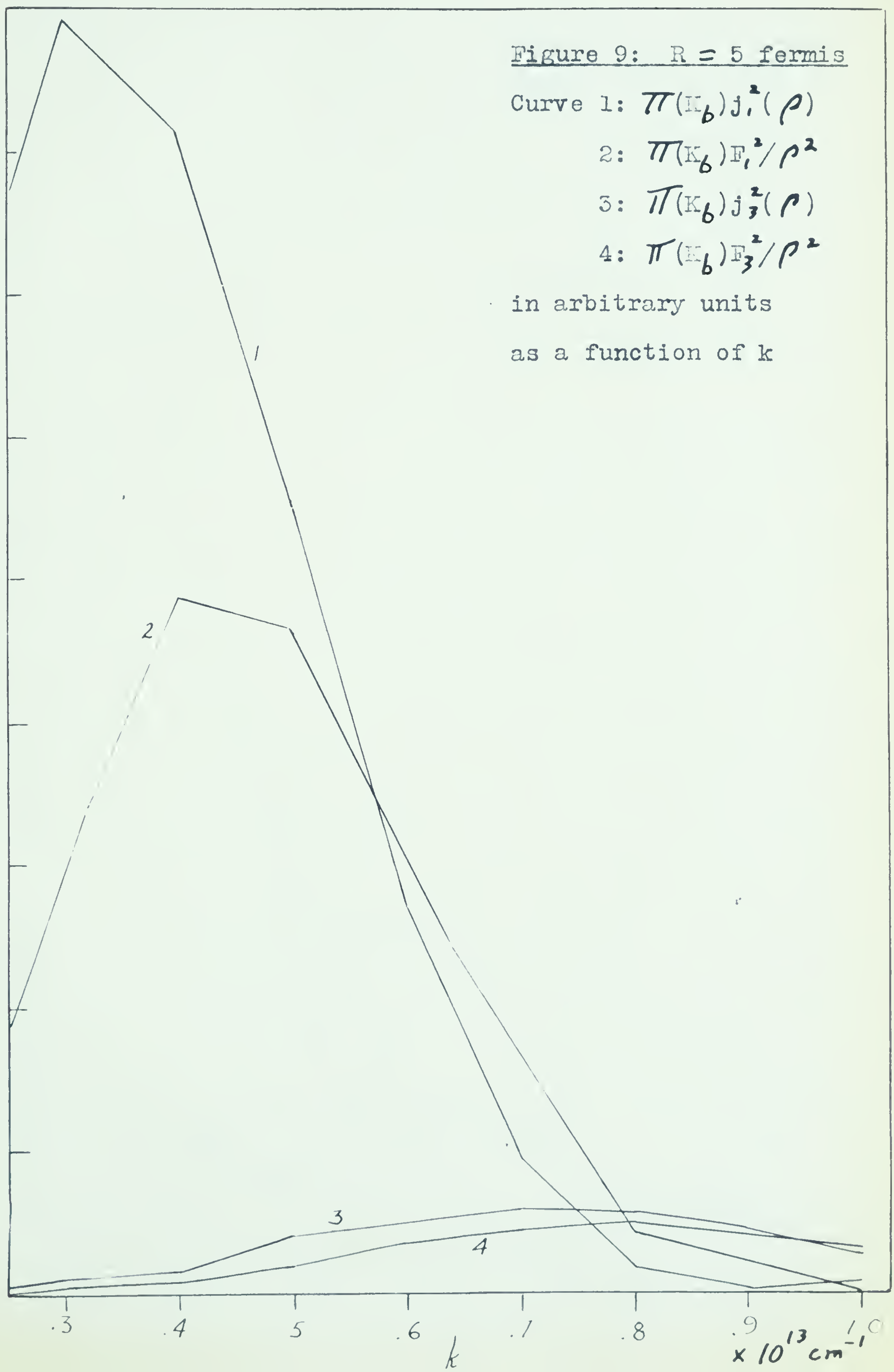
Figure 9: R = 5 fermis
Curve 1: $\pi(K_b) j_1^2(\rho)$
2: $\pi(K_b) F_1^2/\rho^2$
3: $\pi(K_b) j_3^2(\rho)$
4: $\pi(K_b) F_3^2/\rho^2$
in arbitrary units
as a function of k
1
2
3
4
8
7
6
5
4
3
2
1
.3
.4
.5
.6
.7
.8
.9
1.0
k
$\times 10^{13}$ cm^{-1}

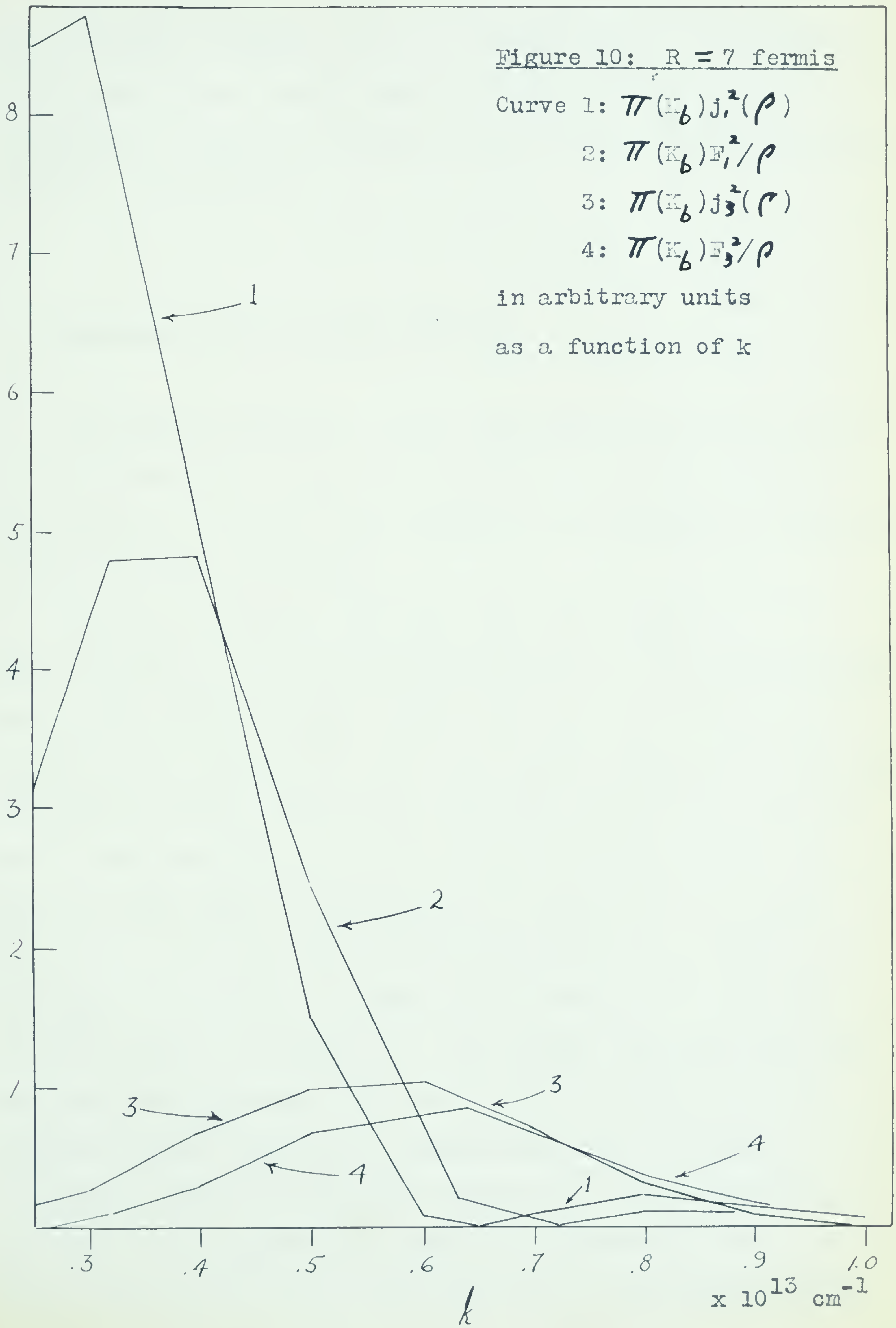
Figure 10: R = 7 fermis
Curve 1: $\pi(K_b) j_1^2(\rho)$
2: $\pi(K_b) F_1^2/\rho$
3: $\pi(K_b) j_3^2(\rho)$
4: $\pi(K_b) F_3^2/\rho$
in arbitrary units
as a function of k
1
2
3
4
8
7
6
5
4
3
2
1
.3
.4
.5
.6
.7
.8
.9
1.0
k
x 10^{13} cm^{-1}

Thus it seems that there really is something different from deuteron stripping, with or without Coulomb corrections, taking place here. Besides heavy-particle stripping, a combination of compound nucleus effects with deuteron stripping remains a possibility.

B- Conclusions about Coulomb Effects

The method suggested for the inclusion of Coulomb effects in (d,n) stripping has therefore not explained the anomalous distributions of the B^{11}(d,n) reaction. Nevertheless it remains of some interest in itself.

The modifications introduced into the angular distributions by this procedure can be seen in the accompanying graphs. The functions F_ℓ^2/ρ^2 start from zero at $\rho = 0$, rise to a first peak, decrease to zero, rise again and again to smaller and smaller peaks, just as the functions $j_\ell^2(\rho)$ do, but they reach each peak later, i.e. at a higher ρ-value, than the corresponding $j_\ell^2(\rho)$; and because of the factor ρ^2 in the denominator, the peaks are neither as high nor as sharp; the area under the curve is pushed out to greater ρ. This peak shift is larger for larger η, the Coulomb functions becoming more and more different from their $\eta = 0$ limiting case the spherical Bessel functions. Remembering that at a given bombarding energy the physical range of θ from 0° to 180° restricts us to looking at a fixed corresponding range of k (e.g. from k = .27 to k = 1.01 for the B^{11} reaction at 2 Mev;

see Figure 6), we can readily describe qualitatively the general effects of these modifications on angular distributions: decrease in total cross-section, decrease in sharpness of the pattern's features, shift of the pattern towards higher θ.

This shift in pattern is obviously more considerable for larger η, hence for larger Z. It can also be seen that it decreases with increasing bombarding energy. For one thing η is inversely proportional to k. But more important is the fact that it is a unique shift, in the curve of σ as a function of k, corresponding to a fixed k-interval; but a fixed k-interval corresponds to a smaller and smaller θ-interval as the bombarding energy increases, as a consequence of Equation P1. Hence this shift in the pattern behaves quite reasonably.

In fact, these modifications are just what is expected on the basis of qualitative arguments already presented, and what is found by vastly more complicated calculations for (d,p) reactions.

It should therefore be interesting to test the predictions of this easy-to-apply partial correction (considering that some tables and graphs of $F_\ell(\rho)$ are now available, (32, 40)) against the more elaborate and difficult-to-compute Coulomb formulas developed for (d,p) reactions, and against experiment in cases where, unlike the case presently studied, pure deuteron stripping still seems to prevail, modified by Coulomb forces but not spoiled beyond recognition by other processes. Not only because of the enormous advantage in computational simplicity of course, but because, for (d,n) reactions, the

methods which consider Coulomb effects on the deuteron as a whole are certainly open to criticism, as pointed out in the beginning of Section N.

S- The Compound Nucleus Process

There remains the possibility of a significant contribution from processes involving compound nucleus formation, in the B^{11} (d,n)C^{12*} reaction. This has always been considered unlikely because there are no levels of C^{13} in the energy region reached by these experiments. (The nearest are two levels of unknown properties at excitations corresponding to 2.18 and 3.08 Mev bombarding energy for the B^{11} + d reaction (Ajzenberg-Selove and Lauritsen, 41).) For the same reason it would be difficult to calculate the angular distributions and correlations to be expected on this basis, since we are neither on a resonance nor in a region where many closely-spaced levels contribute statistically. The only information that seems well established in such a situation is that angular distributions should be symmetrical† about $\theta = 90°$ (Wolfenstein, 42). It is also generally accepted that the distributions should not exhibit features as sharp as those characteristic of direct reactions.

Ward and Grant (43), having obtained at a bombarding energy of .6 Mev an angular distribution that looked roughly symmetrical about 90°, had interpreted it by compound nucleus

† see footnote, page 81.

theory, with reservations. More accurate and extensive measurements have since invalidated this.

A general treatment should include both compound nucleus and direct interaction contributions, possibly with interference between them. Thomas (45) has begun to explore the theory of such messy situations, but this necessarily involves even more loose ends than the work reviewed in Chapter 2, especially since the conditions are not favorable to an unambiguous treatment of either of these two processes.

If it were simply a matter of adding without interference a deuteron stripping cross-section and a cross-section symmetrical about 90° but otherwise undetermined, one might try a very rough analysis of the experimental angular distributions in the following way. Starting with a smoothed-out version of the experimental curves, attribute the whole $\theta > 90^\circ$ part of the distribution to compound nucleus formation; a corresponding part of the distribution for $\theta < 90^\circ$ can then be similarly assigned to compound nucleus formation because of the rule concerning symmetry about 90°; finally the remainder of the distribution for $\theta < 90^\circ$ can be checked for consistency with deuteron stripping, and the part assigned to compound nucleus formation examined to determine its plausibility.

Unfortunately even this much over-simplified program can not be carried out because data are not available for $\theta > 145^\circ$. Hence the program fails also for $\theta < 35^\circ$, and it is not possible to decide whether a segment of curve ranging only from 35° to 90° is part of a stripping curve or not. Moreover the

curve for $\frac{1}{2}$ Mev bombarding energy shows a minimum around 60° and a rise from 60° to 100°, which can not be dealt with on these assumptions. No conclusions can be drawn from this concerning the importance of compound nucleus effects.

Qualitatively, an explanation along those lines would require such effects to be of the same order of magnitude as the deuteron stripping contribution or larger. Ignoring interference,† the predicted compound nucleus angular distribution would have to be fairly isotropic for bombarding energies of $\frac{1}{2}$ and 1 Mev; and to show a minimum at 90° with a gradual rise on each side to about $1\frac{1}{4}$ or $1\frac{1}{2}$ times the minimum ordinate at $\theta \approx 30^\circ$ and 150°, for bombarding energies of $1\frac{1}{2}$ and 2 Mev.

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† In some cases there may be terms antisymmetric about $\theta = 90^\circ$, due to interference between two levels of opposite parity. This possibility is not included in the rough treatment of this section.

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